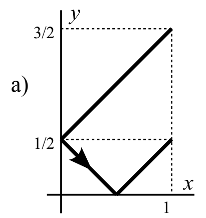
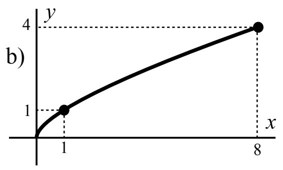
1. a) $\left(-t\,,\tfrac{1}{2}-t\right),\ t\in[-1,0]\,;\ \left(t,\tfrac{1}{2}-t\right),\ t\in[0,\tfrac{1}{2}]\,;\ \left(t,t-\tfrac{1}{2}\right),\ t\in[\tfrac{1}{2},1]$

$\to\ L=\int_{-1}^{0}\sqrt{2}\,dt+\int_0^{1/2}\sqrt{2}\,dt+\int_{1/2}^{1}\sqrt{2}\,dt=2\sqrt{2}$ (¡claro!).

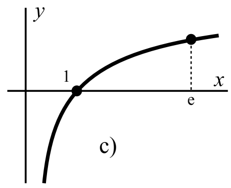


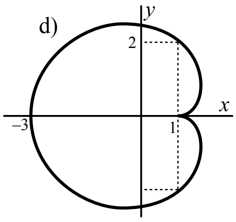


b) $L=\int_1^{e}\frac{\sqrt{x^2+1}}{x}\,dx\overset{u=x+\sqrt{x^2+1}}{=\cdots=}\sqrt{x^2+1}-\log\frac{1+\sqrt{x^2+1}}{x}\Big]_1^{e}\approx 2.0035\,.$

c) $L=\int_1^{8}\sqrt{1+\frac{4}{9x^{2/3}}}\,dx$ es más larga, pero parametrizando $x=y^{3/2}$,

$y\in[1,4]\,,\ L=\int_1^{4}\sqrt{1+\tfrac{9}{4}y}\,dy=\frac{8}{27}\left[10^{3/2}-\tfrac{1}{8}13^{3/2}\right]\approx 7.63\,.$





d) $\|2(\operatorname{sen}2t-\operatorname{sen}t,\cos t-\cos 2t)\|=2\sqrt{2}\,\sqrt{1-\cos t}=4\left|\operatorname{sen}\tfrac{t}{2}\right|,$

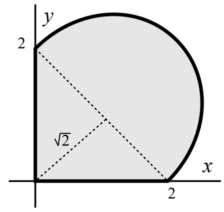
$L=8\int_0^{\pi}\operatorname{sen}\tfrac{t}{2}\,dt=-16\cos\tfrac{t}{2}\Big]_0^{\pi}=16\,.$

2. $r^2=2r\cos\theta+2r\operatorname{sen}\theta\,,\ x^2+y^2=2x+2y\,,\ (x-1)^2+(y-1)^2=2$, circunferencia.

Pasando de integrales: a) $A=\frac{\pi(\sqrt{2})^2}{2}+\frac{2\cdot 1}{2}=\pi+2\,;$ b) $L=2+2+\frac{2\pi\sqrt{2}}{2}=4+\pi\sqrt{2}\,.$

Con integrales: a) $\int_0^{\pi/2}\int_0^{2(\cos\theta+\operatorname{sen}\theta)}r\,dr\,d\theta=\int_0^{\pi/2}(2+4\operatorname{sen}\theta\cos\theta)\,d\theta=\pi+2\,.$

b) $r^2+(r')^2=8\ \to\ L=2+2+\int_0^{\pi/2}\sqrt{8}\,d\theta=4+\pi\sqrt{2}\,.$

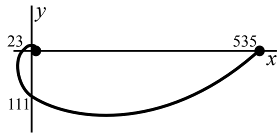


3. a) $f(x,y,z)=yz\,,\ \mathbf{c}(t)=(t,3t,2t)\,,\ \mathbf{c}'(t)=(1,3,2)\,;\ \int_{\mathbf{c}}f\,ds=\int_1^{3}6\,t^2\sqrt{14}\,dt=2\sqrt{14}\,t^3\Big]_1^{3}=52\sqrt{14}\,.$

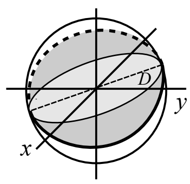
b) $f(x,y,z)=x+z\,,\ \mathbf{c}(t)=\left(t,t^2,\tfrac{2}{3}t^3\right),\ \mathbf{c}'(t)=(1,2t,2t^2)\,;\ \int_{\mathbf{c}}f\,ds=\int_0^{1}\left(t+\tfrac{2}{3}t^3\right)(1+2t^2)\,dt=\tfrac{1}{2}\left(t+\tfrac{2}{3}t^3\right)^2\Big]_0^{1}=\tfrac{25}{18}\,.$

4. $L=\int_0^{2\pi}\sqrt{e^{2\theta}+e^{2\theta}}\,d\theta=\sqrt{2}\int_0^{2\pi}e^{\theta}d\theta=\sqrt{2}\left[e^{2\pi}-1\right]\approx 756\,.$

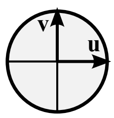
$T_{\text{media}}=\frac{1}{L}\int_0^{2\pi}e^{\theta}\sqrt{e^{2\theta}+e^{2\theta}}\,d\theta=\frac{\sqrt{2}}{L}\int_0^{2\pi}e^{2\theta}d\theta=\frac{\sqrt{2}}{2L}\left[e^{4\pi}-1\right]=\tfrac{1}{2}\left[e^{2\pi}+1\right]\approx 268\,.$



5. La intersección de $x^2+y^2+z^2=1$ y $x+y+z=0$ es una circunferencia C parametrizable con $\mathbf{c}(t)=\cos t\,\mathbf{u}+\operatorname{sen}t\,\mathbf{v}$, siendo $\mathbf{u}\,,\mathbf{v}$ vectores ortogonales unitarios contenidos en el plano.

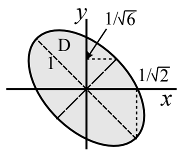


Por ejemplo, $\mathbf{u}=\left(\tfrac{1}{\sqrt{2}},-\tfrac{1}{\sqrt{2}},0\right),\ \mathbf{v}=\left(\tfrac{1}{\sqrt{6}},\tfrac{2}{\sqrt{6}},-\tfrac{1}{\sqrt{6}}\right).\ \|\mathbf{c}'(t)\|=1\,.$



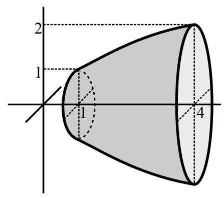
$M=\int_0^{2\pi}\left(\frac{\cos t}{\sqrt{2}}+\frac{\operatorname{sen}t}{\sqrt{6}}\right)^2dt=\int_0^{2\pi}\left(\frac{\cos^2 t}{2}+\frac{\operatorname{sen}t\cos t}{\sqrt{3}}+\frac{\operatorname{sen}^2 t}{6}\right)^2dt=\frac{\pi}{2}+\frac{\pi}{6}=\frac{2\pi}{3}\,.$

[La proyección de C sobre $z=0$ es $2x^2+2y^2+2xy=1$; de ella salen otras parametrizaciones con cálculos bastante más largos, por ejemplo: $\left(x,\tfrac{1}{2}[-x\pm\sqrt{2-3x^2}\,],\tfrac{1}{2}[x\pm\sqrt{2-3x^2}\,]\right)$,
o parametrizando distinto la elipse: $\left(\tfrac{\sqrt{2}}{\sqrt{3}}\operatorname{sen}t,\tfrac{1}{\sqrt{2}}\cos t-\tfrac{1}{\sqrt{6}}\operatorname{sen}t,(\tfrac{1}{\sqrt{6}}-\tfrac{\sqrt{2}}{\sqrt{3}})\operatorname{sen}t-\tfrac{1}{\sqrt{2}}\cos t\right)$].



6. a) $(y^2,y)\,,\ y\in[1,2]$ ó $(x,x^{1/2})\,,\ x\in[1,4]$. Mejor con la primera:

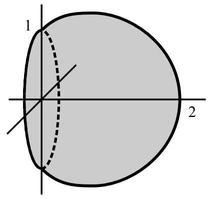
$A=2\pi\int_1^{2}y\sqrt{1+4y^2}\,dy=\frac{\pi}{6}\left(1+4y^2\right)^{3/2}\Big]_1^{2}=\frac{\pi}{6}\left(17^{3/2}-5^{3/2}\right)\approx 30.8\,.$



b) $r=(1+\cos\theta)\,,\ \theta\in\left[0,\tfrac{\pi}{2}\right],\ r'(\theta)=-\operatorname{sen}\theta\,,\ y=(1+\cos\theta)\operatorname{sen}\theta\,,$

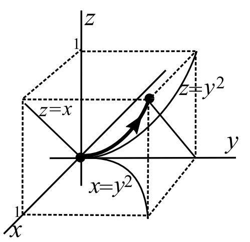
$\|\mathbf{c}'(\theta)\|=\sqrt{r^2+(r')^2}=\sqrt{1+2\cos\theta+\cos^2\theta+\operatorname{sen}^2\theta}=\sqrt{2+2\cos\theta}\,.$

$A=2\sqrt{2}\,\pi\int_0^{\pi/2}(1+\cos\theta)^{3/2}\operatorname{sen}\theta\,d\theta=-\frac{4\sqrt{2}\,\pi}{5}(1+\cos\theta)^{5/2}\Big]_0^{\pi/2}=\frac{4\pi}{5}\left(8-\sqrt{2}\right)\approx 16.55$



7. $\boxed{\mathbf{g}(x,y,z)=\left(3,z^2-1,2yz\right)}\,,\ \mathbf{c}(t)=\left(t^2,t,t^2\right).\ \operatorname{div}\mathbf{g}=\boxed{2y}\,.\ \operatorname{rot}\mathbf{g}=\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\ \partial/\partial x&\partial/\partial y&\partial/\partial z\\ 3&z^2-1&2yz\end{vmatrix}=\boxed{\mathbf{0}}\,.$

$\|\mathbf{c}'\|=\sqrt{1+8t^2}\,,\ \operatorname{div}\bar{g}\,(\mathbf{c}(t))=2t\,.\ \int_{\mathbf{c}}\operatorname{div}\mathbf{g}\,ds=\int_0^{1}2t\sqrt{1+8t^2}\,dt=\frac{1}{12}\left(1+8t^2\right)^{3/2}\Big]_0^{1}=\boxed{\frac{13}{6}}\,.$



$\operatorname{rot}\mathbf{g}=\bar{0}\,,\ \mathbf{g}\in C^1\Rightarrow$ hay U .
$U_x=3\ \to\ U=3x+p(y,z)$
$U_y=z^2-1\to U=yz^2-y+q(x,z)$, $U=3x-y+yz^2\Rightarrow$
$U_z=2yz\ \to\ U=yz^2+r(x,y)$
$\int_{\mathbf{c}}\mathbf{g}\cdot d\mathbf{s}=U(1,1,1)-U(0,0,0)=\boxed{3}\ \forall\mathbf{c}\,.$

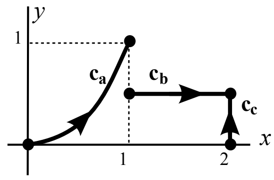
O con la **c** dada: $\int_{\bar{c}}\mathbf{g}\cdot d\mathbf{s}=\int_0^{1}\left(3,t^4-1,2t^3\right)\cdot(2t,1,2t)\,dt=\int_0^{1}\left(6t+5t^4-1\right)dt=3t^2+t^5\Big]_0^{1}-1=\boxed{3}\,.$

O por este camino: $\bar{c}_*(t)=(t,t,t)\,,\ t\in[0,1]\to\int_{\mathbf{c}_*}\mathbf{g}\cdot d\mathbf{s}=\int_0^{1}\left(3,t^2-1,2t^2\right)\cdot(1,1,1)\,dt=\int_0^{1}\left(2+3t^2\right)dt=\boxed{3}\,.$

8. a) $\mathbf{c}(x)=(x,x^2)$, $0\le x\le 1$, $\int_C (x^2+y^2)dx+dy=\int_0^1(x^2+x^4+2x)\,dx=\frac{23}{15}$.

b) $\mathbf{c}(x)=\left(x,\frac12\right)$, $1\le x\le 2$, $\int_C (x^2+y^2)dx+dy=\int_1^2(x^2+\frac14)\,dx=\frac{31}{12}$.

c) $\mathbf{c}(y)=(2,y)$, $0\le y\le \frac12$, $\int_C (x^2+y^2)dx+dy=\int_0^{1/2}dy=\frac12$.

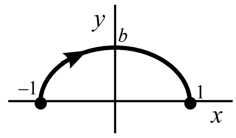


9. $\mathbf{c}(t)=(\cos t, b\,\mathrm{sen}\,t)$, $t\in[0,\pi]$, recorre $b^2x^2+y^2=b^2$ en sentido opuesto.

$T(b)=-\int_0^{\pi}(3b^2\,\mathrm{sen}^2t+2,16\cos t)\cdot(-\mathrm{sen}\,t,b\cos t)\,dt=4b^2-8b\pi+4$. $T'(b)=8(b-\pi)$

$\quad\| \qquad\qquad\qquad\qquad\qquad\qquad\qquad\downarrow$

$3b^2\,\mathrm{sen}\,t(1-\cos^2t)-8b(1+\cos 2t)+2\,\mathrm{sen}\,t \qquad\qquad T$ mínimo si $b=\pi$.



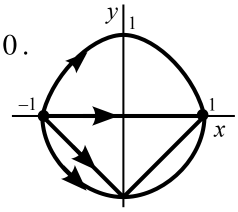
10. $\mathbf{f}(x,y)=(xy,0)$ entre $(-1,0)$ y $(1,0)$. Usamos en todas el parámetro x, $x\in[-1,1]$:

a) $\mathbf{c}(x)=(x,0)$, $\mathbf{c}'=(1,0)$, $\int_{-1}^1 0\,dx=0$. b) $\mathbf{c}(x)=(x,1-x^2)$, $\mathbf{c}'=(1,-2x)$. $\int_{-1}^1(x-x^3)\,dx=0$.

c) $\mathbf{c}(x)=(x,|x|-1)$, $\mathbf{c}'=\begin{cases}(1,-1)\,,\ x<0\\(1,1)\,,\ x>0\end{cases}$. $\int_{-1}^0(-x^2-x)\,dx+\int_0^1(x^2-x)\,dx=0$.

d) $\mathbf{c}(x)=\left(x,-\sqrt{1-x^2}\right)$, $\mathbf{c}'=(1,x(1-x^2)^{-1/2})$. $\int_{-1}^1 -x\sqrt{1-x^2}\,dx=0$.

Pero no es gradiente de nigún campo U pues $f_y=x\ne 0=g_x$ (sobre otros caminos no se anula).

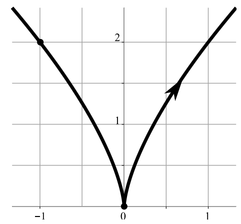


11. a) $\mathbf{c}(t)=(t^3,2t^2)$. $\mathbf{c}'(t)=(3t^2,4t)$, $\|\mathbf{c}'(t)\|=\sqrt{9t^4+16t^2}=|t|\sqrt{9t^2+16}$.

Longitud $L=\int_{-1}^0(-t)\sqrt{9t^2+16}\,dt=\frac{1}{27}\left[-(9t^2+16)^{3/2}\right]_{-1}^0=\frac{5^3-4^3}{27}=\frac{61}{27}$.

b) $h(x,y)=\mathrm{e}^{2x+y}$, $\int_{\mathbf{c}}\nabla h\cdot d\mathbf{s}=h(1,2)-h(0,0)=\mathrm{e}^4-1=\int_0^1(6t^2+4t)\,\mathrm{e}^{2t^3+2t^2}dt=\mathrm{e}^{2t^3+2t^2}\Big]_0^1$

(cálculo innecesario)



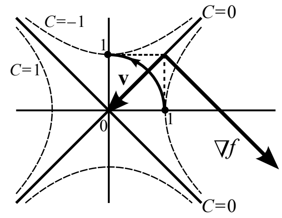
12. **a]** $f(x,y)=x^2-y^2=0\ \to$ las rectas $y=\pm x$. [El resto, hipérbolas]. $\nabla f(x,y)=(2x,-2y)$,

$\nabla f(1,1)=(2,-2)$. $D_{\mathbf{u}}f(1,1)=(2,-2)\cdot(-1,-1)=0$. $\Delta f(x,y)=f_{xx}+f_{yy}=2-2=0$.

b] Como $\mathbf{g}=(2x,-2y)=\nabla f$, será $\int_{\mathbf{c}}\mathbf{g}\cdot d\mathbf{s}=f(0,1)-f(1,0)=-2$. Directamente:

$\mathbf{c}(t)=(\cos t,\mathrm{sen}\,t)$, $t\in\left[0,\frac{\pi}{2}\right]$, $\int_{\mathbf{c}}\mathbf{g}\cdot d\mathbf{s}=\int_0^{\pi/2}(2c,-2s)\cdot(-s,c)\,dt=-2\,\mathrm{sen}^2t\Big]_0^{\pi/2}=-2$.

O bien: $\mathbf{c}(t)=\left(t,\sqrt{1-t^2}\right)$, $t\in[1,0]$, $\int_{\mathbf{c}}\mathbf{g}\cdot d\mathbf{s}=\int_1^0\left(2t,-2\sqrt{\cdot}\right)\cdot\left(1,\frac{-t}{\sqrt{\cdot}}\right)dt=\int_1^0 4t\,dt=-2$.



13. $\mathbf{f}(x,y)=\left(1,x^2\right)$, $x^2+(y-1)^2=1$. Con la parametrización más sencilla $\mathbf{c}(t)=(\cos t,1+\mathrm{sen}\,t)$, $t\in[-\pi,\pi]$ es:

$$\int_{-\pi}^{\pi}\left(1,\cos^2t\right)\cdot(-\mathrm{sen}\,t,\cos t)\,dt=\int_{-\pi}^{\pi}\left[c\left(1-s^2\right)-s\right]dt=\left[\mathrm{sen}\,t+\cos t-\tfrac13\,\mathrm{sen}^3t\right]_{-\pi}^{\pi}=0.$$

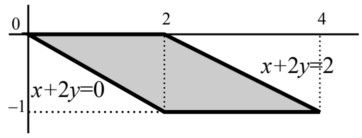
Que por un camino cerrado concreto se anule no implica que sea conservativo. Y no lo es: $g_x-f_y=2x\ne 0$.

[Usando el teorema de Green, la integral pedida pasaría a ser la de la impar $2x$ sobre el círculo simétrico y podríamos asegurar que se anula sin hacer ninguna integral].

14. D limitado por $y=-2$, $y=0$, $x+2y=0$ y $x+2y=2$.

a) $\iint_D(x+2y)\,dx\,dy=\int_{-1}^0\int_{-2y}^{2-2y}(x+2y)\,dx\,dy=\int_{-1}^0\left(\left[\frac{x^2}{2}\right]_{-2y}^{2-2y}+4y\right)dy=\int_{-1}^0 2\,dy=2$.

O bien: $\begin{matrix}u=x+2y\,,\ x=u-2v\\ v=y\end{matrix}$, $\frac{\partial(x,y)}{\partial(u,v)}=\begin{vmatrix}1&-2\\0&1\end{vmatrix}=1\ \to\ \int_0^2\int_{-1}^0 u\,dv\,du=\int_0^2 u\,du=2$.



b) Como $\mathbf{f}(x,y)=(1,\cos y)$ cumple $(1)_y=0=(\cos y)_x\ \Rightarrow$ deriva de potencial ($U=x+\mathrm{sen}\,y$) $\Rightarrow \oint_{\partial D}\mathbf{f}\cdot d\mathbf{s}=0$.

Directamente (largo): $\mathbf{c_1}=\left(t,-\frac{t}{2}\right)$, $t\in[0,2]$, $\mathbf{c_2}=(t,-1)$, $t\in[2,4]$, $\mathbf{c_3}=\left(t,1-\frac{t}{2}\right)$, $t\in[4,2]$, $\mathbf{c_4}=(t,0)$, $t\in[2,0]$,

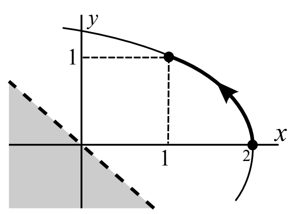
$$\int_0^2\left[1-\tfrac12\cos\tfrac{t}{2}\right]dt+\int_2^4 dt+\int_4^2\left[1-\tfrac12\cos(1-\tfrac{t}{2})\right]dt+\int_2^0 dt=-\tfrac14\,\mathrm{sen}\,1+\tfrac14\,\mathrm{sen}\,1=0.$$

15. Para $\mathbf{f}(x,y)=\left(-\frac{y}{(y+x)^2},\frac{x}{(y+x)^2}\right)$ es $f_y=\frac{y-x}{(y+x)^3}=g_x$, y hallamos un potencial en $y+x>0$:

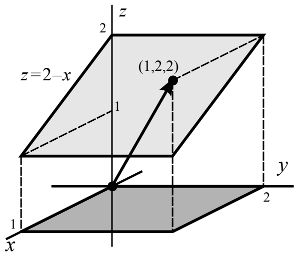
$U=\int\frac{-y\,dx}{(y+x)^2}=\frac{y}{y+x}+p(y)$

$U=\int\frac{x\,dy}{(y+x)^2}=\frac{y}{y+x}-1+q(x)$ $\quad\Rightarrow\ U=\frac{y}{y+x}\ \Rightarrow\int_{\mathbf{c}}\mathbf{f}\cdot d\mathbf{s}=U(1,1)-U(2,0)=\frac12-0=\frac12$.

Directamente: $\mathbf{c}=\left(2-y^2,y\right)$, $y\in[0,1]$, $\int_{\mathbf{c}}\mathbf{f}\cdot d\mathbf{s}=\int_0^1\frac{2+y^2}{(2+y-y^2)^2}\,dy=\cdots=\frac{y}{2+y-y^2}\Big]_0^1=\frac12$.



16. $g(x,y,z)=y\,e^{2x-z}$. a) En $[0,1]\times[0,2]$ es $z=2-x > z=0$ (no se precisa el dibujo).



$$\iiint_V g=\int_0^2\!\int_0^1\!\int_0^{2-x} y\,e^{2x-z}\,dz\,dx\,dy = 2\int_0^1\left[e^{2x}-e^{3x-2}\right]dx=\left[e^{2x}-\tfrac{2}{3}e^{3x-2}\right]_0^1=\frac{3e^2-3+2e+2e^{-2}}{3}\,.$$

b) i) $\mathbf{c}(t)=(t,2t,2t)$, $\mathbf{c}'=(1,2,2)$, $\|\mathbf{c}'\|=3$. $\int_{\mathbf{c}} g\cdot ds=\int_0^1 g(\mathbf{c}(t))\|\mathbf{c}'(t)\|\,dt=\int_0^1 6t\,dt=3$.

ii) Más fácil: $\int_{\mathbf{c}}\nabla g\cdot d\mathbf{s}=g(1,2,2)-g(0,0,0)=2$.

Directamente: $\nabla g=\left(2y\,e^{2x-z},e^{2x-z},-ye^{2x-z}\right)$, $\int_0^1(4t,1,-2t)\cdot(1,2,2)\,dt=\int_0^1 2\,dt=2$.

17. $\mathbf{f}(x,y,z)=(xy\,,2x\,,-yz)$. $\operatorname{div}\mathbf{f}=y+0-y=\boxed{0}$. $\operatorname{rot}\mathbf{f}=\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\ \partial/\partial x&\partial/\partial y&\partial/\partial z\\ xy&2x&-yz\end{vmatrix}=-z\,\mathbf{i}+0\,\mathbf{j}+(2-x)\,\mathbf{k}=\boxed{(-z\,,0\,,2-x)}$.

[no deriva de un potencial].

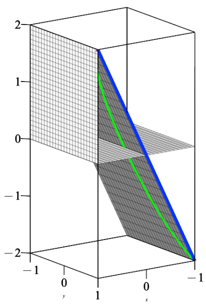
$$\int_{\mathbf{c}}\mathbf{f}\cdot d\bar{s}=\int_0^{\pi}(cs,2c,-s)\cdot(-s,c,0)\,dt=\int_0^{\pi}(2\cos^2 t-\operatorname{sen}^2 t\cos t)\,dt=\pi+\left[\tfrac12\operatorname{sen}2t-\tfrac13\operatorname{sen}^3 t\right]_0^{\pi}=\boxed{\pi}\,.$$

Como $\|\mathbf{c}'\|=\sqrt{\operatorname{sen}^2 t+\cos^2 t+0}=1$, la longitud de la curva es $L=\int_0^{\pi}1\,dt=\boxed{\pi}$.

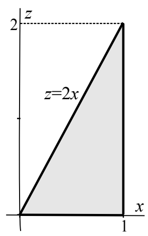
18. $\mathbf{F}(x,y,z)=(x,y,z)$. a) $\mathbf{c}(t)=(t,t,t)$, $t\in[0,1]$; $\int_{\mathbf{c}}\mathbf{F}\cdot d\mathbf{s}=\int_0^1(t,t,t)\cdot(1,1,1)\,dt=\int_0^1 3t\,dt=\frac32$.

b) $\mathbf{c}(t)=(\cos t,\operatorname{sen}t,0)$, $t\in[0,2\pi]$; $\int_{\mathbf{c}}\mathbf{F}\cdot d\mathbf{s}=\int_0^{2\pi}(\cos t,\operatorname{sen}t,0)\cdot(-\operatorname{sen}t,\cos t,0)\,dt=\int_0^{2\pi}0\,dt=0$.

19. $\mathbf{F}(x,y,z)=\left(2xz+y,x,x^2\right)$. a) $\operatorname{div}\mathbf{F}=2z$, $\operatorname{rot}\mathbf{F}=\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\ \partial_x&\partial_y&\partial_z\\ 2xz+y&x&x^2\end{vmatrix}=(0,2x-2x,1-1)=\mathbf{0}$ y $\mathbf{F}\in C^1\Rightarrow\exists U$ potencial.



b) $\iiint_V 2z\,dz\,dy\,dx=\int_0^1\!\int_{-1}^1\!\int_0^{2x}2z\,dz\,dy\,dx=\int_0^1 8x^2\,dx=\boxed{\tfrac83}$.



c) Hallando el potencial U: $\begin{array}{l}U=x^2z+xy+\cdots\\ U=xy+\cdots\\ U=x^2z+\cdots\end{array}\Rightarrow U(x,y,z)=x^2z+xy$.

Por tanto: $\int_{\mathbf{c}}\mathbf{F}\cdot d\bar{s}=U(1,1,2)-U(-1,1,-2)=3-(-3)=\boxed{6}$.

Como es conservativo y la integral no depende de camino, se podría ir por el segmento que une los puntos:

$\mathbf{c}_*(t)=(t,1,2t)$, $t\in[-1,1]$. $\int_{-1}^1\left(4t^2+1,t,t^2\right)\cdot(1,0,2)\,dt=2\int_0^1(6t^2+1)\,dt=\boxed{6}$.

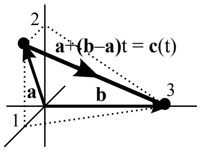
Usando la definición con la $\mathbf{c}$ dada se complica el cálculo y surge una integral que parece no calculable:

$$\int_{-1}^1\left(4t^2+e^{t^2-1},t,t^2\right)\cdot\left(1,2te^{t^2-1},2\right)dt=2\int_0^1\left[6t^2+(2t^2+1)\,e^{t^2-1}\right]dt=\left[2t^3+te^{t^2-1}\right]_0^1=\boxed{6}\,.$$

derivada de $\uparrow te^{t^2-1}$

d) $\mathbf{c}(1)=(1,1,2)$, $\mathbf{c}'(1)=(1,2,2)$. $\mathbf{x}=(1+t,1+2t,2+2t)$ corta $z=0$ en $(0,-1,0)$

20. $\mathbf{F}(x,y,z)=(1,2yz,y^2)$, $\mathbf{c}(t)=(1,0,2)+t(-1,3,-2)=(1-t,3t,2-2t)$, $t\in[0,1]$.



$\int_{\mathbf{c}}\mathbf{F}\cdot d\mathbf{s}=\int_0^1\left(1,12t-12t^2,9t^2\right)\cdot(-1,3,-2)\,dt=\int_0^1(-1+36t-54t^2)\,dt=-1$.

Como $\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\ \partial/\partial x&\partial/\partial y&\partial/\partial z\\ 1&2yz&y^2\end{vmatrix}=(2y-2y)\,\mathbf{i}+0\,\mathbf{j}+0\,\mathbf{k}=\mathbf{0}$ y $\mathbf{F}\in C^1(\mathbf{R}^3)$ hay potencial y la integral será -1 para toda curva.

$\begin{array}{l}U=x+p(y,z)\\ U=y^2z+q(x,z)\\ U=y^2z+r(x,y)\end{array}\Rightarrow U=x+y^2z$, $\int_{\mathbf{c}}\mathbf{F}\cdot d\mathbf{s}=U(0,3,0)-U(1,0,2)=0-1=-1$ (de otra forma).

21. $\mathbf{f}(x,y,z)=\left(z^2,2y,cxz\right)$. a) $\operatorname{div}\mathbf{f}=2+cx$. $\operatorname{rot}\mathbf{f}=\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\ \partial/\partial x&\partial/\partial y&\partial/\partial z\\ z^2&2y&cxz\end{vmatrix}=(0\,,2z-cz\,,0)$.

b) Para $c=2$ es $\operatorname{rot}\mathbf{f}=\mathbf{0}$ y como $\mathbf{f}\in C^1(\mathbf{R}^3)$, existe el potencial: $\begin{array}{l}U_x=z^2\to U=xz^2+p(y,z)\\ U_y=2y\to U=y^2+q(x,z)\\ U_z=2xz\to U=xz^2+r(x,y)\end{array}$, $U=xz^2+y^2$.

c) Por tanto, $\int_{\mathbf{c}}\mathbf{f}\cdot d\mathbf{s}=U(1,0,1)-U(0,0,0)=1$, sin necesidad de hacer ninguna integral de línea.

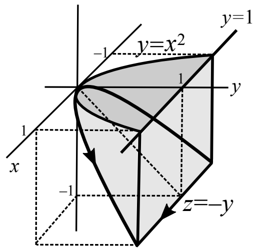
d) Una parametrización: $\mathbf{c}(t)=(t,0,t)$, $t\in[0,1]\to\int_{\mathbf{c}}\mathbf{f}\cdot d\mathbf{s}=\int_0^1(t^2,0,ct^2)\cdot(1,0,1)\,dt=\int_0^1(c+1)t^2dt=\frac{c+1}{3}$.

22. $\mathbf{f}(x,y,z)=\left(\mathrm{e}^{-z},1,-x\mathrm{e}^{-z}\right)$. a) $\mathbf{c}(t)=(1,1-t,3t)$, $t\in[0,1]\to$

$$\int_{\mathbf{c}}\mathbf{f}\cdot d\mathbf{s}=\int_0^1\left(\mathrm{e}^{-3t},1,-\mathrm{e}^{-3t}\right)\cdot(0,-1,3)\,dt=\int_0^1\left(-1-3\mathrm{e}^{-3t}\right)dt=\boxed{\mathrm{e}^{-3}-2}.$$

b) $\operatorname{rot}\mathbf{f}=\mathbf{0}$, $\mathbf{f}\in C^1\Rightarrow$ hay potencial. $\begin{array}{l}U=x\mathrm{e}^{-z}+p(y,z)\\U=y+q(x,z)\\U=x\mathrm{e}^{-z}+r(x,y)\end{array}$, $\boxed{U=x\mathrm{e}^{-z}+y}$ $\left[\int_{\mathbf{c}}\mathbf{f}\cdot d\mathbf{s}=U(1,0,3)-U(1,1,0)=\mathrm{e}^{-3}-2\right]$.

23. a) $\iiint_V y=\int_{-1}^1\int_{x^2}^1\int_{-y}^0 y\,dz\,dy\,dx=\int_{-1}^1\int_{x^2}^1 y^2\,dy\,dx=\frac{2}{3}\int_0^1\left(1-x^6\right)dx=\frac{2}{3}\left[1-\frac{1}{7}\right]=\boxed{\frac{4}{7}}.$



O bien: $\iiint_V y=\int_0^1\int_{-y}^0\int_{-\sqrt{y}}^{\sqrt{y}} y\,dx\,dz\,dy=\int_0^1\int_{-y}^0 2y^{3/2}\,dz\,dy=\int_0^1 2y^{5/2}dy=\boxed{\frac{4}{7}}.$

b) $\operatorname{rot}\mathbf{f}=\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\\partial/\partial x&\partial/\partial y&\partial/\partial z\\y&x&z\end{vmatrix}=(0,0,1-1)=\mathbf{0}$ y $\mathbf{f}\in C^1$ en $\mathbf{R}^3\Rightarrow$ existe función potencial.

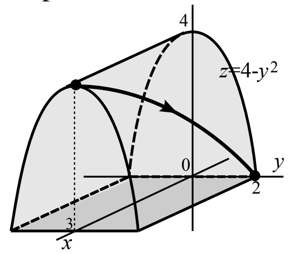
$\begin{array}{l}U_x=y\to U=xy+p(y,z)\\U_y=x\to U=xy+q(x,z)\\U_z=z\to U=\frac{1}{2}z^2+r(x,y)\end{array}$, $\boxed{U=xy+\frac{1}{2}z^2}\Rightarrow\int_{\mathbf{c}}\mathbf{f}\cdot d\mathbf{s}=U(1,1,-1)-U(-1,1,-1)=\boxed{2}$ para todo camino.

O bien: $\mathbf{c}(x)=\left(x,x^2,-x^2\right)$, $x\in[-1,1]\to\int_{\mathbf{c}}\mathbf{f}\cdot d\mathbf{s}=\int_{-1}^1\left(x^2,x,-x^2\right)\cdot(1,2x,-2x)\,dt=\int_{-1}^1\left(3x^2+2x^3\right)dx=\boxed{2}.$

O por el camino más simple: $\mathbf{c}(x)=(x,1,-1)$, $x\in[-1,1]\to\int_{\mathbf{c}}\mathbf{f}\cdot d\mathbf{s}=\int_{-1}^1(1,x,-1)\cdot(1,0,0)\,dx=\int_{-1}^1 dx=\boxed{2}.$

24. a) $\operatorname{div}\mathbf{f}=2x-2$. $\operatorname{rot}\mathbf{f}=\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\\partial/\partial x&\partial/\partial y&\partial/\partial z\\y^2&xy&-2z\end{vmatrix}=0\,\mathbf{i}+0\,\mathbf{j}+(2y-2y)\,\mathbf{k}=(0,0,0)\overset{\mathbf{f}\in C^1}{\Longrightarrow}\mathbf{f}$ deriva de un potencial.

$\nabla(\operatorname{div}\mathbf{f})=(2,0,0)$. $\quad\Delta(\mathbf{f}\cdot\mathbf{f})=\Delta\left(y^4+4x^2y^2+4z^2\right)=20y^2+8x^2+8$.



b) $z=4-y^2$ corta $z=0$ en las rectas $y=\pm2$. V es el del dibujo. Por tanto:

$$\iiint_V\operatorname{div}\mathbf{f}=\int_0^3\int_{-2}^2\int_0^{4-y^2}(2x-2)\,dz\,dy\,dx=\int_0^3(2x-2)\,dx\int_{-2}^2\left(4-y^2\right)dy=32.$$

c) Por ser conservativo, podemos hallar la integral hallando el potencial U:

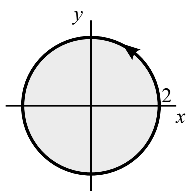
$\begin{array}{l}U_x=y^2\to U=xy^2+p(y,z)\\U_y=2xy\to U=xy^2+q(x,z)\\U_z=-2z\to U=-z^2+r(x,y)\end{array}$, $U=xy^2-z^2\Rightarrow\int_{\mathbf{c}}\mathbf{f}\cdot d\mathbf{s}=U(0,2,0)-U(3,0,4)=16$.

O directamente: $\int_{\mathbf{c}^*}\mathbf{f}\cdot d\mathbf{s}=\int_0^2\left(t^2,6t-3t^2,2t^2-8\right)\cdot\left(-\frac{3}{2},1,-2t\right)dt=\int_0^2\left(22t-\frac{9}{2}t^2-4t^3\right)dt=11t^2-\frac{3}{2}t^3-t^4\Big]_0^2=16.$

O hallar la integral siguiendo un camino más sencillo, por ejemplo el segmento que une los puntos:

$\mathbf{c}^*(t)=(3-3t,2t,4-4t)$, $t\in[0,1]\to\int_0^1\left(4t^2,12t-12t^2,8t-8\right)\cdot(-3,2,-4)\,dt=\int_0^1\left(32-8t-36t^2\right)dt=16.$

25. $\mathbf{g}(x,y)=\left(1,xy^2\right)$. $g_x-f_y=y^2$. No deriva de un potencial. i) $\mathbf{c}(t)=(2\cos t,2\operatorname{sen}t)$, $t\in[0,2\pi]$.

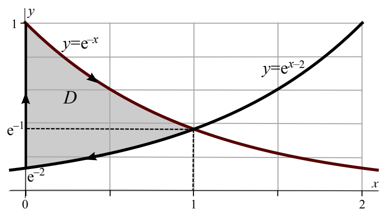


$\to\int_{\mathbf{c}}\mathbf{g}\cdot d\mathbf{s}=\int_0^{2\pi}\left(1,8cs^2\right)\cdot(-2s,2c)\,dt=2\cos t\Big]_0^{2\pi}+\int_0^{2\pi}4\operatorname{sen}^2 2t\,dt=\int_0^{2\pi}(2-2\cos4t)\,dt=4\pi.$

ii) Green: $\iint_\circ y^2\,dx\,dy=\int_0^{2\pi}\int_0^2 r^3\operatorname{sen}^2\theta\,dr\,d\theta=\left[\frac{1}{4}r^4\right]_0^2\cdot\frac{1}{2}\int_0^{2\pi}(1-\cos2\theta)\,d\theta=4\pi.$

$\Big[$En cartesianas mucho más largo: $\int_{-2}^2\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}y^2\,dy\,dx=\frac{2}{3}\int_{-2}^2\left(4-x^2\right)^{3/2}dx=\cdots\Big]$.

26. a) $\int_0^1\int_{\mathrm{e}^{x-2}}^{\mathrm{e}^{-x}}x\,\mathrm{e}^x\,dy\,dx=\int_0^1\left[x-x\mathrm{e}^{2x-2}\right]dx=\frac{x^2}{2}-\frac{x}{2}\mathrm{e}^{2x-2}\Big]_0^1+\frac{1}{2}\int_0^1\mathrm{e}^{2x-2}dx=\frac{1-\mathrm{e}^{-2}}{4}.$



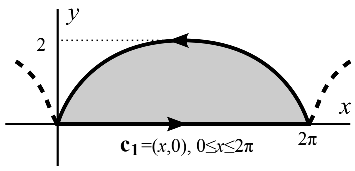
b) $g_x-f_y=-x\,\mathrm{e}^x$. Según Green, la $\oint_{\partial D}\mathbf{f}\cdot d\mathbf{s}$ vale lo de arriba. Directamente:

$\mathbf{c}_1(t)=(0,t)$, $y\in\left[\mathrm{e}^{-2},1\right]$; $\mathbf{c}_2(t)=\left(t,\mathrm{e}^{-t}\right)$, $t\in[0,1]$; $\mathbf{c}_3(t)=\left(t,\mathrm{e}^{t-2}\right)$, $t\in[1,0]$.

$$\int_{\partial D}\mathbf{f}\cdot d\mathbf{s}=\int_{\mathrm{e}^{-2}}^1(0,1)\cdot(0,1)\,dt+\int_0^1(t,1)\cdot\left(1,-\mathrm{e}^{-t}\right)dt+\int_1^0\left(t\mathrm{e}^{2t-2},1\right)\cdot\left(1,\mathrm{e}^{t-2}\right)dt$$

$$=1-\mathrm{e}^{-2}+\int_0^1\left(t-\mathrm{e}^{-t}\right)dt+\int_1^0\left(t\mathrm{e}^{2t-2}+\mathrm{e}^{t-2}\right)dt=\frac{3}{2}-\mathrm{e}^{-2}+\left[\mathrm{e}^{-t}\right]_0^1+\left[\frac{t}{2}\mathrm{e}^{2t-2}-\frac{1}{4}\mathrm{e}^{2t-2}+\mathrm{e}^{t-2}\right]_1^0=\frac{1-\mathrm{e}^{-2}}{4}.$$

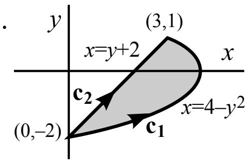
27. $x=t-\operatorname{sen}t$, $y=1-\cos t$, $0\le t\le2\pi$. Utilizando Green:



$$A=\frac{1}{2}\oint_{\partial D}x\,dy-y\,dx=\frac{1}{2}\int_0^{2\pi}0-\frac{1}{2}\int_0^{2\pi}\left[(t-\operatorname{sen}t)\operatorname{sen}t-(1-\cos t)^2\right]d\theta$$

$$=\int_0^{2\pi}\left(1-\cos t-\frac{1}{2}t\operatorname{sen}t\right)dt=3\pi.$$

28. a) $\mathbf{f}(x,y)=(y^2,2x)$, $\mathbf{c_1}=(4-t^2,t)$, $t\in[-2,1]$, $\mathbf{c_2}=(t,t-2)$, $t\in[0,3]$ (sentido opuesto).

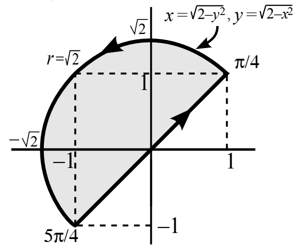


$$\int_{\partial D}\mathbf{f}\cdot d\mathbf{s}=\int_{-2}^{1}(-2t^3+8-2t^2)\,dt-\int_0^3(t^2-2t+4)\,dt=26-\tfrac12-12=\tfrac{27}{2}.$$

$$g_x-f_y=2-2y\,,\ \int_{-2}^{1}\int_{y+2}^{4-y^2}(2-2y)\,dx\,dy=\int_{-2}^{1}(4-6y+2y^3)\,dy=\tfrac{27}{2}.$$

b) $\mathbf{f}(x,y)=(y^2,xy)$. $\iint_D -y\,dx\,dy=-\int_{\pi/4}^{5\pi/4}\int_0^{\sqrt2}r^2\,\mathrm{sen}\,\theta=\left[\tfrac13 r^3\right]_0^{\sqrt2}\left[\cos\theta\right]_{\pi/4}^{5\pi/4}=\tfrac{2\sqrt2}{3}\left[-\tfrac{1}{\sqrt2}-\tfrac{1}{\sqrt2}\right]=-\tfrac43$.

En cartesianas (de las dos formas) es más complicado. Por ejemplo:



$$\iint_D f=-\int_{-\sqrt2}^{-1}\int_{-\sqrt{2-x^2}}^{\sqrt{2-x^2}}y\,dy\,dx-\int_{-1}^{1}\int_{-x}^{\sqrt{2-x^2}}y\,dy\,dx=0+\int_{-1}^{1}\left[x^2-1\right]dx=-\tfrac43.$$

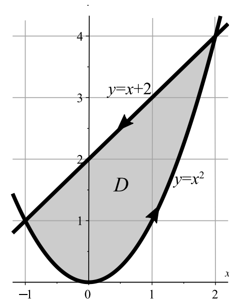
Parametrizaciones sencillas: $\mathbf{c_1}(t)=(t,t)$, $t\in[-1,1]$ (en sentido correcto).

$\mathbf{c_2}(t)=(\sqrt2\cos t,\sqrt2\,\mathrm{sen}\,t)$, $t\in\left[\tfrac{\pi}{4},\tfrac{5\pi}{4}\right]$ (también en buen sentido).

$$\oint_{\partial D}\mathbf{f}\cdot d\mathbf{s}=\int_{-1}^{1}(t^2,t^2)\cdot(1,1)\,dt+\int_{\pi/4}^{5\pi/4}(2\,\mathrm{sen}^2t,2\,\mathrm{sen}\,t\cos t)\cdot(-\sqrt2\,\mathrm{sen}\,t,\sqrt2\cos t)$$

$$=\int_{-1}^{1}2t^2\,dt+2\sqrt2\int_{\pi/4}^{5\pi/4}\left[sc^2-s^3\right]dt=\tfrac43+2\sqrt2\left[\cos t-\tfrac23\cos^3t\right]_{\pi/4}^{5\pi/4}=\tfrac43+2\left[1+1-\tfrac13-\tfrac13\right]=-\tfrac43.$$

c) $\mathbf{f}(x,y)=(-xy,y)$. $g_x-f_y=x$. $\iint_D x=\int_{-1}^{2}\int_{x^2}^{2+x}x\,dy\,dx=\int_{-1}^{2}(2x+x^2-x^3)\,dx=\tfrac94$.



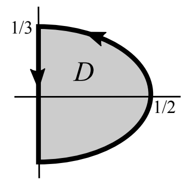
O peor: $\int_0^1\int_{-\sqrt y}^{\sqrt y}x\,dx\,dy+\int_1^4\int_{y-2}^{\sqrt y}x\,dx\,dy=0+\tfrac12\int_1^4(5y-y^2-4)\,dy=\tfrac94$.

Parametrizamos así ∂D: $\mathbf{c}_1(x)=(x,x^2)$, $x\in[-1,2]$. $\mathbf{c}_2(x)=(x,x+2)$, $x\in[2,-1]$

$$\oint_{\partial D}\mathbf{f}\cdot d\mathbf{s}=\int_{-1}^{2}(-x^3,x^2)\cdot(1,2x)\,dx+\int_{2}^{-1}(-x^2-2x,2+x)\cdot(1,1)\,dx$$

$$=\int_{-1}^{2}x^3\,dx+\int_{-1}^{2}(x^2+x-2)\,dx=\tfrac{15}{4}+\tfrac93+\tfrac32-6=\tfrac{15}{4}-\tfrac32=\tfrac94.$$

d) $\mathbf{f}(x,y)=(x,x^2)$. $g_x-f_y=2x$. $\iint_D 2x=\int_{-1/3}^{1/3}\int_0^{\sqrt{1-9y^2}/2}2x\,dx\,dy=\tfrac12\int_0^{1/3}(1-9y^2)\,dy=\tfrac19$.

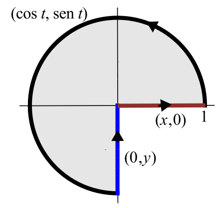


O con $x=\tfrac r2\cos\theta$, $y=\tfrac r3\,\mathrm{sen}\,\theta$, $J=\tfrac r6$, $\tfrac16\int_0^1\int_{-\pi/2}^{\pi/2}r^2\cos\theta\,d\theta\,dr=\tfrac16\cdot\tfrac12\cdot2=\tfrac19$.

Sobre $x=0$ es $\mathbf{f}=\mathbf{0}$ y la elipse viene dada por: $\mathbf{c}(t)=\left(\tfrac12\cos t,\tfrac13\,\mathrm{sen}\,t\right)$, $t\in\left[-\tfrac{\pi}{2},\tfrac{\pi}{2}\right]$:

$$\to\ \oint_{\partial D}\bar{f}\cdot d\bar{s}=\int_{-\pi/2}^{\pi/2}\left(\tfrac c2,\tfrac{c^2}{4}\right)\cdot\left(-\tfrac s2,\tfrac c3\right)dt=\tfrac16\int_0^{\pi/2}(1-s^2)c\,dt=\tfrac16\left[s-\tfrac13s^3\right]_0^{\pi/2}=\tfrac19.$$

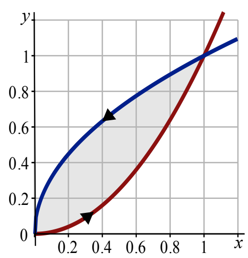
e) $\mathbf{f}(x,y)=(xy,2-x^2)$, $g_x-f_y=-3x$. $\iint_D -3x=-\int_0^{3\pi/2}\int_0^1 3r^2\cos\theta\,dr\,d\theta=1$.



$$\oint_{\partial D}\mathbf{f}\cdot d\mathbf{s}=\int_0^{3\pi/2}(\cos t\,\mathrm{sen}\,t,2-\cos^2t)\cdot(-\mathrm{sen}\,t,\cos t)\,dt+\int_{-1}^{0}(0,2)\cdot(0,1)\,dy+\int_0^1 0\,dx$$

$$=\int_0^{3\pi/2}\cos t\,(2-\cos^2t-\mathrm{sen}^2t)\,dt+2=\mathrm{sen}\,t\Big]_0^{3\pi/2}+2=1.$$

f) $\mathbf{f}(x,y)=(x^3,x^2y)$. $g_x-f_y=2xy$. $\int_0^1\int_{x^2}^{\sqrt x}2xy\,dy\,dx=\int_0^1x(x-x^4)\,dx=\tfrac13-\tfrac16=\tfrac16$.

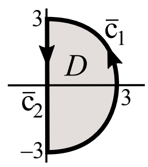


Parametrizamos las curvas: $\bar{c}_1=(x,x^2)$, $x\in[0,1]$. $\bar{c}_2=(y^2,y)$, $y\in[1,0]$.

$$\oint_{\partial D}\mathbf{f}\cdot d\mathbf{s}=\int_{\mathbf{c}_1}\mathbf{f}\cdot d\mathbf{s}+\int_{\mathbf{c}_2}\mathbf{f}\cdot d\mathbf{s}=\int_0^1(x^3,x^4)\cdot(1,2x)\,dx-\int_0^1(y^6,y^5)\cdot(2y,1)\,dy$$

$$=\int_0^1(x^3+2x^5)\,dx-\int_0^1(2y^7+y^5)\,dy=\tfrac16.$$

g) $\mathbf{f}(x,y)=(y^2,x^2)$. $g_x-f_y=2(x-y)$. $2\int_{-\pi/2}^{\pi/2}\int_0^3 r^2(\underbrace{\cos\theta}_{\text{par}}-\underbrace{\mathrm{sen}\,\theta}_{\text{impar}})\,dr\,d\theta=36\int_0^{\pi/2}\cos\theta\,d\theta=36$.



O bien: $2\int_0^3\int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}}(x-y)\,dy\,dx=4\int_0^3x(9-x^2)^{1/2}dx=-\tfrac43(9-x^2)^{3/2}\Big]_0^3=\tfrac{4\cdot27}{3}=36$.

O bien: $\int_{-3}^{3}\int_0^{\sqrt{9-y^2}}(2x-2y)\,dx\,dy=\int_{-3}^{3}(9-y^2-\underbrace{2y\sqrt{9-y^2}}_{\text{impar}})\,dy=54-2\left[\tfrac{y^3}{3}\right]_0^3=54-18=36$.

La ∂D. Semicircunferencia: $\bar{c}_1=(3\cos t,3\,\mathrm{sen}\,t)$, $t\in\left[-\tfrac{\pi}{2},\tfrac{\pi}{2}\right]$. Segmento: $(0,y)$, $y\in[3,-3]$.

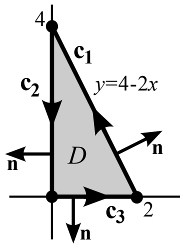
$$\int_{\bar{c}_1}\mathbf{f}\cdot d\mathbf{s}=27\int_{-\pi/2}^{\pi/2}\underbrace{(s^2,c^2)\cdot(-s,c)}_{c^3-s^3\text{ (par–impar)}}\,dt=54\int_0^{\pi/2}\underbrace{\cos^3t}_{c-s^2c}\,dt=54\left[\mathrm{sen}\,t-\tfrac13\,\mathrm{sen}^3t\right]_0^{\pi/2}=36.$$

[En cartesianas: $\bar{c}_*=(\sqrt{9-y^2},y)$, $y\in[-3,3]$. $\int_{\bar{c}_*}\mathbf{f}\cdot d\mathbf{s}=\int_{-3}^{3}(y^2,9-y^2)\cdot\left(\tfrac{-y}{\sqrt{\cdot}},1\right)dx=2\int_0^3\left(9-y^2-\underbrace{\tfrac{y^3}{\sqrt{\cdot}}}_{\text{impar}}\right)dy$].

Para el segmento: $\int_{\bar{c}_2}\mathbf{f}\cdot d\mathbf{s}=\int_3^{-3}\underbrace{(y^2,0)\cdot(0,1)}_{=0}\,dy=0$. Por tanto, $\oint_{\partial D}\mathbf{f}\cdot d\mathbf{s}=\boxed{36}$, como la doble.

29. $\mathbf{g}(x,y)=\left(x^2,-2xy\right)$. **Green**. $\iint_D (g_x-f_y)=\int_0^2\int_0^{4-2x}(-2y)\,dy\,dx=-\int_0^2 (4-2x)^2\,dx=\frac{1}{6}(4-2x)^3\Big]_0^2=-\frac{32}{3}$.

La ∂D está formada por 3 segmentos fáciles de parametrizar. Por ejemplo:



$\mathbf{c_1}(t)=(t,4-2t)$, $t\in[2,0]$ [para ir en sentido antihorario].

$\mathbf{c_2}(t)=(0,t)$, $t\in[4,0]$. $\mathbf{c_3}(t)=(t,0)$, $t\in[0,2]$. Entonces:

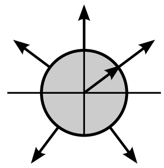
$$\oint_{\partial D}\mathbf{g}\cdot d\mathbf{s}=\int_{\mathbf{c_1}}+\int_{\mathbf{c_2}}+\int_{\mathbf{c_3}}=\int_4^0\left(t^2,4t^2-8t\right)\cdot(1,-2)\,dt+\int_4^0 0\,dt+\int_0^2\left(t^2,0\right)\cdot(1,0)\,dt$$

$$=\int_2^0(16t-7t^2)\,dt+0+\int_0^2 t^2\,dt=-32+\frac{56}{3}+\frac{8}{3}=-\frac{32}{3}.$$

Divergencia. $\iint_D \operatorname{div}\mathbf{f}\,dx\,dy=\iint_D(2x-2x)\,dx\,dy=0$. Las normales unitarias exteriores $\mathbf{n}$ a cada segmento son, respectivamente: $\left(\frac{2}{\sqrt5},\frac{1}{\sqrt5}\right)$, $(-1,0)$ y $(0,-1)$. Sus normas $\|\mathbf{c}'_{\mathbf{k}}(t)\|$ son: $\sqrt5$, 1 y 1. Por tanto:

$$\oint_{\partial D}\mathbf{g}\cdot\mathbf{n}\,ds=\int_0^4\left(t^2,4t^2-8t\right)\cdot(2,1)\,dt+\int_0^4 0\,dt+\int_0^2\left(t^2,0\right)\cdot(0,-1)\,dt=\int_0^2(6t^2-8t)\,dt-\int_0^2 0\,dt=16-16=0.$$

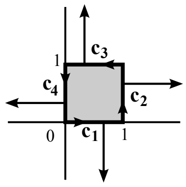
30. i) $\mathbf{f}(x,y)=(x,y)$, $D=\{x^2+y^2\leq 1\}$. $\mathbf{f}\cdot\mathbf{n}=1$ (dos vectores unitarios en el mismo sentido).

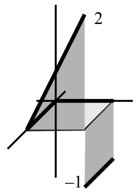


$$\oint_{\partial D}1\,ds\underset{\uparrow}{=}2\pi=\iint_D 2\,dx\,dy\underset{\uparrow}{=}2\pi.$$

longitud   2 veces el área

ii) $\mathbf{f}(x,y)=(2xy,-y^2)$, $\operatorname{div}\mathbf{f}=0$, $\iint_D\operatorname{div}\mathbf{f}=0$.





Los cuatro $\mathbf{n}$ exteriores son: $\mathbf{n_1}=(0,-1)$, $\mathbf{n_2}=(1,0)$, $\mathbf{n_3}=(0,1)$, $\mathbf{n_4}=(-1,0)$.

Luego: $\oint_{\partial D}\mathbf{f}\cdot\mathbf{n}\,ds=\int_{\mathbf{c_1}}y^2ds+\int_{\mathbf{c_2}}2xy\,ds-\int_{\mathbf{c_3}}y^2ds-\int_{\mathbf{c_4}}2xy\,ds$

$=0+\int_0^1 2y\,dy-\int_0^1 1\,dx-0=0$, tomando $\mathbf{c_2}=(1,y)$, $y\in[0,1]$, $\mathbf{c_3}=(1-x,1)$, $x\in[0,1]$.

[Hasta aquí el control 2].