junio MMII - 11/12

Problema 1

```
> e1:=diff(u(x,y),y)-2*y*diff(u(x,y),x)=4*x*y:s1:=2*x*y^2+y^4:
  [pdsolve(e1,u(x,y)),eval(subs(u(x,y)=s1,e1)),subs(y=-1,s1)];
```

$$\left[u(x,y) = -x^2 + \_F1\left(x + y^2\right), 4\,x\,y = 4\,x\,y, 2\,x + 1\right] \quad (1)$$

Problema 2

```
> es:=3*x*diff(y(x),x$2)+diff(y(x),x)+x*y(x):
  dsolve(es,y(x));rhs(dsolve(es,y(x),series));
```

$$y(x) = \_C1\,x^{1/3}\,\mathrm{BesselJ}\left(\frac{1}{3}, \frac{1}{3}\sqrt{3}\,x\right) + \_C2\,x^{1/3}\,\mathrm{BesselY}\left(\frac{1}{3}, \frac{1}{3}\sqrt{3}\,x\right)$$

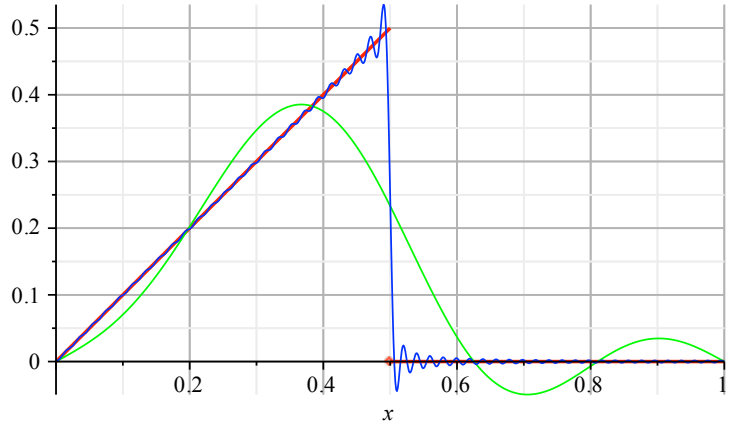
$$\_C1\,x^{2/3}\left(1 - \frac{1}{16}x^2 + \frac{1}{896}x^4 + \mathrm{O}\left(x^6\right)\right) + \_C2\left(1 - \frac{1}{8}x^2 + \frac{1}{320}x^4 + \mathrm{O}\left(x^6\right)\right) \quad (2)$$

Problema 3

```
> g:=piecewise(-5/2<x and x<-3/2,x+2,-1/2<x and x<1/2,x,
  3/2<x and x<5/2,x-2):Xn:=sin(n*Pi*x):
  cn:=expand(2*int(x*Xn,x=0..1/2) assuming n::integer);
  S:=k->sum(cn*Xn,n=1..k):S(5);
  plot([g,S(5),S(100)],x=0..1,thickness=[2,1,1],discont=true,
  scaling=constrained,gridlines=true,color=[red,green,blue]);
  [evalf(subs(x=1/2,S(5))),evalf(subs(x=1/2,S(100))),
    sum(1/(2*n-1)^2,n=1..infinity)];
  [evalf(subs(x=1/4,S(5))),evalf(subs(x=1/4,S(100)))];
```

$$cn := \frac{2\sin\left(\frac{1}{2}n\pi\right)}{n^2\pi^2} - \frac{\cos\left(\frac{1}{2}n\pi\right)}{n\pi}$$

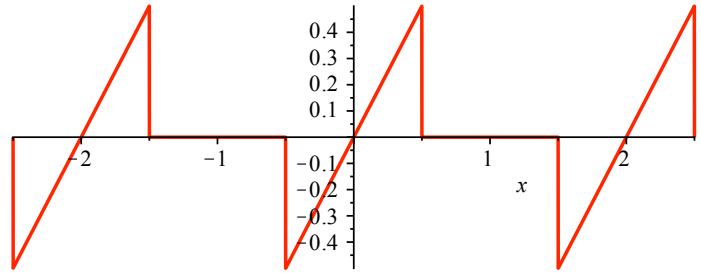
$$\frac{2\sin(\pi x)}{\pi^2} + \frac{1}{2}\frac{\sin(2\pi x)}{\pi} - \frac{2}{9}\frac{\sin(3\pi x)}{\pi^2} - \frac{1}{4}\frac{\sin(4\pi x)}{\pi} + \frac{2}{25}\frac{\sin(5\pi x)}{\pi^2}$$



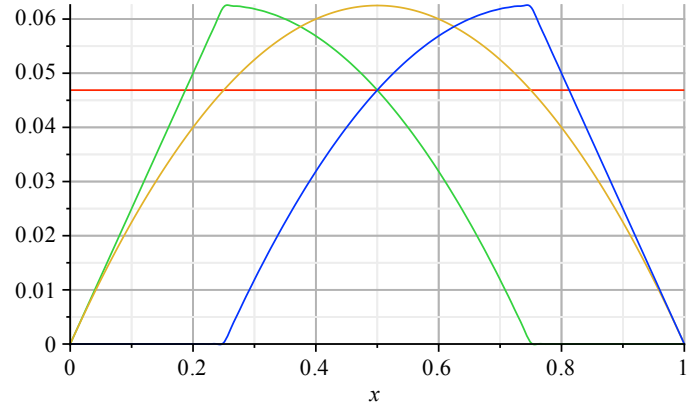
$$\left[0.2332638803, 0.2489868186, \frac{1}{8}\pi^2\right]$$

$$[0.2807920553, 0.2515912000] \quad (3)$$

```
> plot(g,x=-5/2..5/2,thickness=2);
```

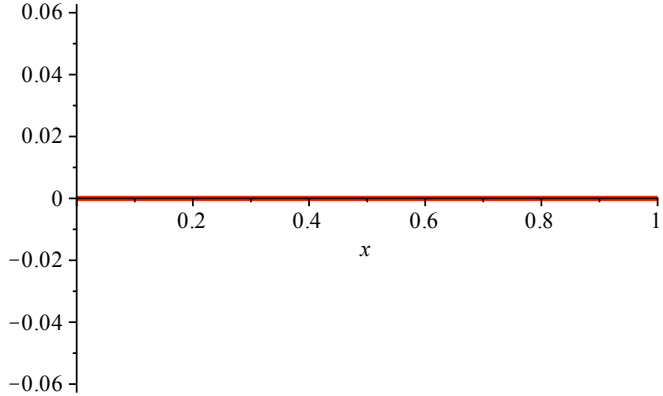



```
> ss:=t->sum(cn/n/Pi*sin(n*Pi*t)*Xn,n=1..100):
  plot([3/64,ss(1/4),ss(1/2),ss(3/4)],x=0..1,gridlines=true);
```




```
> with(plots):sd:=t->1/2*int(g,x=x-t..x+t):
  animate(plot,[sd(t),x=0..1],t=0..2,thickness=3,frames=41);
```

$t = 2.000000000$



Problema 4A

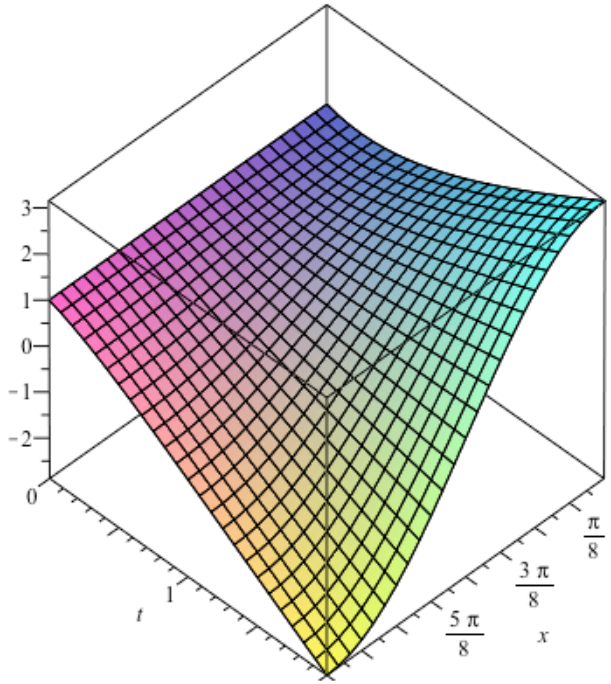
```
> oc:=diff(u(x,t),t)-diff(u(x,t),x$2)+u(x,t):
  dsolve({diff(T(t),t)+T(t)=0,T(0)=1},T(t));
  dsolve({diff(T(t),t)+2*T(t)=4*t,T(0)=0},T(t));
  so:=exp(-t)+(2*t-1+exp(-2*t))*cos(x):
  normal(eval([subs(u(x,t)=so,oc),subs(t=0,so)]));
```

$$T(t) = e^{-t}$$

$$T(t) = 2\,t - 1 + e^{-2\,t}$$

$$[4\cos(x)\,t,\ 1] \qquad (4)$$

```
> plot3d(so,x=0..Pi,t=0..2,axes=boxed);
```



Problema 4B

```
> P:=diff(u(r,t),r$2)+diff(u(r,t),r)/r+diff(u(r,t),t$2)/r^2:
  dsolve(r^2*diff(R(r),r$2)+r*diff(R(r),r)-R(r)=r^2);
  dsolve(r^2*diff(R(r),r$2)+r*diff(R(r),r)-4*R(r)=0);
  sp:=r*(r-2)/3*cos(t)+r^2/4*sin(2*t):
  radsimp(normal(eval([subs(u(r,t)=sp,P),subs(r=2,sp)])));
```

$$R(r) = r\,\_C2 + \frac{1}{3}\,r^2 + \frac{\_C1}{r}$$

$$R(r) = \frac{\_C1}{r^2} + \_C2\,r^2$$

$$[\cos(t),\ \sin(2\,t)] \qquad (5)$$

Problema 5A

```
> F:=diff(u(r,t),r$2)+2*diff(u(r,t),r)/r+diff(u(r,t),t$2)/r^2
    +cos(t)/sin(t)/r^2*diff(u(r,t),t):
  ee:=r^2*diff(R(r),r$2)+2*r*diff(R(r),r)-n*(n+1)*R(r):
  factor(dsolve({ee=0,R(2)=0},R(r)));
  se:=(8/r^2-r)/7*cos(t):
  normal(eval([subs(u(r,t)=se,F),subs(r=1,se),subs(r=2,se)]));
```

$$R(r) = -\_C2\,\left(2\,4^n\,r^{-n-1} - r^n\right)$$

$$[0,\ \cos(t),\ 0] \qquad (6)$$

Problema 5B

```
> ec:=diff(u(x,t),t)-2*diff(u(x,t),x$2)+t*u(x,t)=0:
  sc:=1/sqrt(t+1)*exp(-1/8*x^2/(t+1)-t^2/2):
  normal([eval(subs(u(x,t)=sc,ec)),subs(t=0,sc)]);
```

$$\left[0 = 0,\ e^{-\frac{1}{8}x^2}\right] \qquad (7)$$