Par2M

(segundo parcial de matemáticas)

Problema 1.

```
> g:=(1-exp(x^3))/(x-sin(x)):
  [taylor(numer(%),x),taylor(denom(%),x)];
  [limit(g,x=0),limit(g,x=infinity)];
```

$$\left[x^3 + O(x^6),\ -\frac{1}{6}x^3 + \frac{1}{120}x^5 + O(x^6)\right]$$

$$[-6, -\infty] \qquad (1)$$

Problema 3.

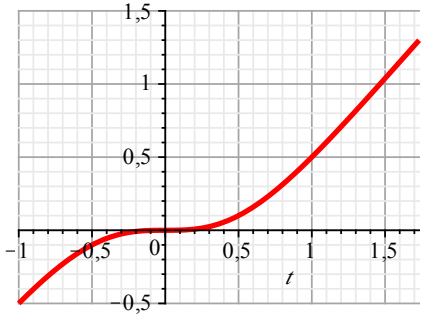
```
> f:=t^3/(1+t^2):F:=int(f,t=-1..tan(x));
  normal(diff(F,x));eval(subs(x=Pi/3,F));
```

$$F := -\frac{1}{2} + \frac{1}{2}\ln(2) + \frac{1}{2}\tan(x)^2 - \frac{1}{2}\ln(1+\tan(x)^2)$$

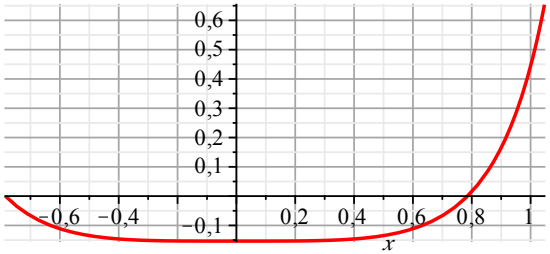
$$\tan(x)^3$$

$$1 - \frac{1}{2}\ln(2) \qquad (2)$$

```
> plot(f,t=-1..sqrt(3),-0.5..1.5,thickness=3);
```




```
> plot(F,x=-Pi/4..Pi/3,thickness=2);
```



Problema 4.

```
> h:=arctan(1/x)/arctan(x^2):int(h,x);
  evalf([int(h,x=0.1..1),int(h,x=1..10000)]);
```

$$\int \frac{\arctan\left(\frac{1}{x}\right)}{\arctan(x^2)}\,dx$$

$$[12.06062122, 6.009487217] \qquad (3)$$

Problema 5.

```
> f:=(9-exp(x))/(exp(2*x)+3):
  factor(simplify(diff(f,x)));solve(%,x);
  eval([subs(x=0,f),subs(x=log(3),f)]);
  int(f,x);int(f,x=0..log(3));
  evalf([%,2*log(3),1/2*log(3)]);
```

$$\frac{e^x\left(e^{2x} - 3 - 18\,e^x\right)}{\left(e^{2x}+3\right)^2}$$

$$\ln\left(9 + 2\sqrt{21}\right),\ \ln\left(9 - 2\sqrt{21}\right)$$

$$\left[2, \frac{1}{2}\right]$$

$$-\frac{3}{2}\ln\left(\left(e^x\right)^2 + 3\right) + 3\ln\left(e^x\right) - \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}e^x\sqrt{3}\right)$$

$$-\frac{1}{18}\sqrt{3}\,\pi + \frac{3}{2}\ln(3)$$

$$[1.345618540, 2.197224578, 0.5493061445] \qquad (4)$$

```
> with(plots):multiple(plot,[f,x=0..4,color=blue],
   [[2,1/2],x=0..log(3)],thickness=2,gridlines=true);
```