Pair annihilation/creation in covariant few-body equations of QFT .

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Introduction

- Lattice QFT

- Continuum QFT
  $\Rightarrow$ Summing infinite number of Feynman diagrams with the help of few-body equations in QFT

Introduction

- $2q2\bar{q}$ covariant equations in QFT
  - Exotic systems: Tetraquark bound states of $2q2\bar{q}$ in the framework of covariant four-body equations
    W. Heupel, G. Eichmann, C. Fischer, arXiv:1206.5129v1(hep-ph) (2012).
  - Description of scattering of usual mesons (i.e. of $q\bar{q}$ bound states)
  - Scattering amplitudes corresponding to all possible processes in the system of two nucleons and two anti-nucleons.
  - Detailed study of non-exotic $q\bar{q}$ bound states: the two-body $q\bar{q}$ BS kernel can be constructed as a solution of the $2q2\bar{q}$ equations, which is equivalent to taking into account of an infinite sum of physically meaningful Feynman diagrams in the $q\bar{q}$ BS kernel.
  - A system of two electrons and two positrons.

Introduction

- Equations for the $2q2\bar{q}$ differ system from the $4q$ equations due to a possibility of $q\bar{q}$ annihilation in the $2q2\bar{q}$ system.

- Major problem: over-counting due to ambiguity of few-body cuts for systems with $N > 3$ and/or where particles can disappear.
  This problem does not exist in two-body and 3q BS equations.
  - Over-counting in 4q equations due to Feynman rules (diagrams not ordered in time):
    DRAWING***
    A. K. & A. Khvedelidze, Theor. Math. Phys. **90**, 62 (1992).
    W. Heupel, G. Eichmann, C. Fischer, arXiv:1206.5129v1(hep-ph) (2012).

  - Over-counting caused by particle creation/absorbtion:
    A. K. & B. Blankleider, Nucl. Phys. **A574**, 788 (1994)

4-body equations without pair annihilation

- The purpose of this talk is to derive covariant equations in QFT for a system of 2 particles + 2 antiparticles

  where pair-annihilation is taken into account

  - It is non-trivial that resultant equations can be arranged so that their kernels are two-body scattering amplitudes (like in the simplest case of Faddeev equations)

- 4-body Green function G and the corresponding t-matrix, T, ($G = G_0 + G_0 T G_0$) satisfies a Dyson equation which relates it to the four-body interaction kernel K via

$$G = G_0 + G_0 K G, \qquad T = K + K G_0 T \tag{1}$$

4-body equations without pair annihilation

- Particles (quarks) labeled 1,2; antiparticles (antiquarks) 3,4. The kernel that involves only two-body correlations is written as the sum of three terms:

$$K = \sum K_{aa'} \tag{2}$$

index $a = 12, 13, 14, 23, 24, 34$, enumerates six possible pairs of particles, the double index, $aa' = (12, 34), (13, 24), (14, 23)$, enumerates three possible two pairs of particles. $K_{aa'}$ describes the part of the four-body kernel where all interactions are switched off except those within the pairs a and a'.

4-body equations without pair annihilation

- To express $K_{aa'}$ in terms of the two-body kernels K_a we use the obvious expression for the Green function, $G_{aa'}$, corresponding to the sum of all Feynman diagrams where the pair a is disconnected from the pair a':

$$G_{aa'} = G_a G_{a'} \tag{3}$$

$G_a = 1 + T_a$ are two-body Green functions. Using definition of $K_{aa'}$,

$$G_{aa'} = G_0 + G_0 K_{aa'} G_{aa'}, \tag{4}$$

one gets unusual pair-interaction approximation:

$$K_{aa'} = K_a + K_{a'} - K_a K_{a'} \tag{5}$$

where

$$G_a = 1 + K_a G_a \tag{6}$$

The product $K_a K_{a'}$ is subtraction term to avoid over-counting in covariant equations

4-body equations with pair annihilation

- qq kernels, K_{12}, K_{34}, are the sum of two-body irreducible diagrams; all of them are connected.

- $q\bar{q}$ kernels, $K_{13}, K_{23}, K_{14}, K_{24}$, contain disconnected parts as well which correspond to the annihilation (creation) of the $q\bar{q}$ pairs into (from) vacuum in the initial (final) states.

4-body equations with pair annihilation

- These disconnected parts in the kernels, K_a, can be derived from the Eq. (6) given that the same disconnectedness is present in G_a in the form of the product of two single quark propagators corresponding to independent propagation of these two quarks in t-channel

- Drawing
  not to be confused with the free part of G_a,

- Drawing
  which corresponds to independent propagation of two quarks in the s-channel. Such disconnected parts appear in the case of the $2q2\bar{q}$ and they cannot be accounted for by equations suitable for the $4q$ system.

4-body equations with pair annihilation

- Derive $2q2\bar{q}$ equations in analogy with the 4q equations. In equations (1-6) difference from the 4q case: $q\bar{q}$ kernel, K_a, contains disconnected part corresponding the annihilation of $q\bar{q}$ pairs into vacuum. Important: these disconnected parts lead to a double counting problem in the "pair interaction approximation", Eq. (2). 3- and 4-body forces contain counter terms which cancel the overcounted terms generated by iteration of the pair interaction kernels.
  Apart from that there are disconnected graphs involving "inelastic" subprocesses; we dealt with them in the case of πNN (Kv. Blankleider, Phys.Lett. B307 (1993) 7)
  We do not deal with 3- and 4-body force and with "inelastic" disconnectedness:
  we work out a part of the pair interaction kernel
  which is physically meaningful on one hand side
  and does not generate double counted terms on another .

4-body equations with pair annihilation

- Using the product property (3), $G_{aa'} = G_a G_{a'}$, and the definition of the two pair interaction t-matrix, $T_{aa'}$,

$$G_{aa'} = G_0 + G_0 T_{aa'} G_0, \tag{7}$$

we get

$$T_{aa'} = T_a + T_{a'} + T_a T_{a'} \tag{8}$$

where $q\bar{q}$ scattering amplitude, $T_a = T_a^C + A_a$, contains disconnected part, A_a corresponding to $q\bar{q}$ annihilation into vacuum,

$$\begin{aligned}
T_{12} &= T_{12}^C \\
T_{34} &= T_{34}^C \\
T_{13} &= T_{13}^C + A_{13} \\
T_{14} &= T_{14}^C + A_{14} \\
T_{23} &= T_{23}^C + A_{23} \\
T_{24} &= T_{24}^C + A_{24}
\end{aligned} \tag{9}$$

4-body equations with pair annihilation

- The proper $q\bar{q}$ scattering amplitude, T_a^C, is the connected part of the $q\bar{q}$ Green function; the superscript "C" will be suppressed below. In the two-body $q\bar{q}$ subspace A_a has the form

$$A_{23}(k_2, k_3, p_2, p_3) = \delta(p_2 - p_3)S^{-1}(k_2)S^{-1}(p_2) \tag{10}$$

where the momenta are assigned to the quark line direction, so that, for example, $p_2(-p_3)$ are the momenta of incoming quark(antiquark).

4-body equations with pair annihilation

•

$$
\begin{aligned}
& G_{23}(k_2,k_3,p_2,p_3) = G^0_{23} + T_{23} + A_{23} \\
& = \int e^{i(k_2y_2-k_3y_3-p_2x_2+p_3x_3)} \langle\langle 0|Tq(y_2)\bar{q}(y_3)\bar{q}(x_2)q(x_3)|0\rangle\rangle dy_2dy_3dx_2dx_3 \\
& G^0_{23}(k_2,k_3,p_2,p_3) = S(p_2)\delta(k_2-p_2)S(-p_3)\delta(k_3-p_3) \\
& = \int e^{i(k_2y_2-p_2x_2)} \langle\langle 0|Tq(y_2)\bar{q}(x_2)|0\rangle\rangle dy_2dx_2 \\
& \times \int e^{i(-k_3y_3+p_3x_3)} \langle\langle 0|Tq(x_3)\bar{q}(y_3)|0\rangle\rangle dy_3dx_3 \\
= \quad & S(p_2)\delta(k_2-p_2)S(-p_3)\delta(k_3-p_3) \\
& A_{23}(k_2,k_3,p_2,p_3) = -S(k_2)\delta(k_2-k_3)S(p_2)\delta(p_2-p_3) \\
& = -\int e^{i(k_2y_2-k_3y_3)} \langle\langle 0|Tq(y_2)\bar{q}(y_3)|0\rangle\rangle dy_2dx_2 \\
& \times \int e^{i(-p_2x_2+p_3x_3)} \langle\langle 0|Tq(x_3)\bar{q}(x_2)|0\rangle\rangle dy_3dx_3 \qquad (11)
\end{aligned}
$$

4-body equations with pair annihilation

- Right from the beginning we discard the products of the disconnected terms, $A_{13}A_{24}$ and $A_{14}A_{23}$, in $T_{aa'}$, as they do not contribute to the physical 4-body t-matrix.
  According Faddeev's trick 4-body t-matrix, T, in the "two pair interaction" approximation (2) would satisfy the following set of equations

$$T = \sum \mathcal{T}_{aa'} \tag{12}$$

  where

$$\mathcal{T}_{aa'} = T_{aa'} + T_{aa'}(\mathcal{T}_{bb'} + \mathcal{T}_{cc'}), \qquad aa' \neq bb' \neq cc' \neq aa'. \tag{13}$$

  These equations suffer double-counting problem if the annihilation terms, A_a, are taken into account via relations (9). Eq. (13) in full (with its double counting problem) is worked out (work in progress),

4-body equations with pair annihilation

- For more clarity we handle Eq. (13) in approximation, close to W. Heupel, G. Eichmann, C. Fischer, arXiv:1206.5129v1(hep-ph) (2012)

$$
\begin{aligned}
T_{12,34} &= T_{12} + T_{34} + T_{12}T_{34} \\
&\to T_{12}T_{34} \\
T_{13,24} &= T_{13} + A_{13} + T_{24} + A_{24} + (T_{13} + A_{13})(T_{24} + A_{24}) \\
&\to A_{13} + A_{24} + T_{13}T_{24} \\
T_{14,23} &= T_{14} + A_{14} + T_{23} + A_{23} + (T_{14} + A_{14})(T_{23} + A_{23}) \\
&\to A_{14} + A_{23} + T_{14}T_{23}
\end{aligned}
\tag{14}
$$

Eq. (14) is based on physically motivated assumption: tetraquark = bound state of two mesons or of diquark-antidiquark pair. Our modification: $q\bar{q}$ pair interaction t-matrices $(T_{13,24}$ and $T_{14,23})$ have disconnected parts, A_a.

4-body equations with pair annihilation

- Analysis of overcounting problem inherent in the Eq. (13) gives an additional support to the approximation (14). Namely, the main part of discarded terms (if not all of them) in the kernels (8-9) cause double counting. For example, the terms like, $T_a A_{a'}$, are discarded in the approximate kernels (14), because the second iteration, $T_{14,23}T_{12,34}T_{14,23}$, contains, $A_{23}T_{12}T_{34}A_{23}$, which is double-counted in the part, $T_{14}A_{23}$, of the kernel $T_{14,23}$ from (8-9).
  Thus instead of introducing three- and four-body forces to compensate double-counted terms the approximation (14) is proposed which is equivalent to accounting for some of these compensating forces without going beyond a pair interaction model

4-body equations with pair annihilation

- Approximation (14) still generates overcounted terms. $A_{13}A_{14}$ generated in the first iteration can be obtained from A_{13} by switching antiquark 34 legs in the initial state, but this term will be produced by antisymmetrisation of the solution of the Eq. (13). Such troublesome double counted terms can be avoided by having modified Eq. (13), namely the modified equation when iterated should not let the kernels A_{13} and A_{14} meet each other

4-body equations with pair annihilation

- To this end we split the kernels (14) into two parts (if time is left, even more general case can be considered)

$$T_{aa'} = A_{aa'} + T^2_{aa'} \tag{15}$$

where

$$\begin{aligned} T^2_{12,34} &= T_{12}T_{34}, & A_{12,34} = 0 \\ T^2_{13,24} &= T_{13}T_{24}, & A_{13,24} = A_{13} + A_{24} \\ T^2_{14,23} &= T_{14}T_{23}, & A_{14,23} = A_{14} + A_{23} \end{aligned} \tag{16}$$

Then the modified equations for the t-matrix are

$$T = \sum_a (\mathcal{T}^A_{aa'} + \mathcal{T}^2_{aa'}) \tag{17}$$

where

$$\begin{aligned} \mathcal{T}^A_{aa'} &= A_{aa'} + A_{aa'}(\mathcal{T}^2_{bb'} + \mathcal{T}^2_{cc'}), & aa' \neq bb' \neq cc' \neq aa'. \\ \mathcal{T}^2_{aa'} &= T^2_{aa'} + T^2_{aa'}(\mathcal{T}_{bb'} + \mathcal{T}_{cc'}). \end{aligned} \tag{18}$$

4-body equations with pair annihilation

- The corresponding bound state equations are

$$\Psi = \sum_a (\Psi^A_{aa'} + \Psi^2_{aa'}) \tag{19}$$

where

$$\begin{aligned} \Psi^A_{aa'} &= A_{aa'}(\Psi^2_{bb'} + \Psi^2_{cc'}), & aa' \neq bb' \neq cc' \neq aa'. \\ \Psi^2_{aa'} &= T^2_{aa'}(\Psi_{bb'} + \Psi_{cc'}). \end{aligned} \tag{20}$$

From the Eq. (20) one has

$$\begin{aligned} \Psi^2_{aa'} &= T^2_{aa'}\left(\Psi^2_{bb'} + \Psi^2_{cc'} + A_{bb'}[\Psi^2_{aa'} + \Psi^2_{cc'}] + A_{cc'}[\Psi^2_{bb'} + \Psi^2_{aa'}]\right), \qquad aa' \neq bb' \neq c \\ &= T^2_{aa'}\left([1 + A_{cc'}]\Psi^2_{bb'} + [1 + A_{bb'}]\Psi^2_{cc'} + [A_{bb'} + A_{cc'}]\Psi^2_{aa'}\right). \end{aligned}$$

The kernels $T^2_{aa'}$ are not compact as they contain singular δ-functions, corresponding to the conservation of the total 4-momentum of each pair, a and a' . Kernels $T^2_{aa'} A_{cc'}$ are not compact either as they involve δ-function restricting the total momentum of some $q\bar{q}$ pairs to zero.

4-body equations with pair annihilation

- One should iterate the Eq. (21) once to cast it in the form where the kernels are compact. The procedure of compactification is simpler if one uses the separable approximation for two body t-matrices, $T_a = \Gamma_a D_a \bar{\Gamma}_a$, e.g.,

$$T_{12}(p_1'p_2', p_1p_2) = -\Gamma(p_1'p_2')D(P)\bar{\Gamma}(p_1p_2), \qquad P = p_1 + p_2 \tag{22}$$

DRAWING

4-body equations with pair annihilation

- leading according Eq. (20) to the factorization of the bound state wave function, $\Psi^2_{aa'} = \Gamma_a D_a \Gamma_{a'} D_{a'} \Phi_{aa'}$,

$$\Psi^2_{aa'}(p,q,q',P) = \Gamma_a(q,Q) D_a(Q) \bar{\Gamma}_{a'}(q',Q') D_{a'}(Q') \Phi_{aa'}(p,P) \tag{23}$$

where $\Phi_{aa'}(p,P)$ are the components of the $2q2\bar{q}$ bound state in the MM and $D\bar{D}$ space. P is the bound state total momentum, p is the relative momentum between its respective constituents, q, q' are the relative momenta of the (anti-)diquarks and mesons, Q, Q' are their off-mass-shell momenta.
DRAWING
So using the ansatz, (22, 23) in Eq. (21) one gets the set of equations for $\Phi_{aa'}$,

$$\begin{aligned}
\Phi_{aa'} &= \bar{\Gamma}_a \bar{\Gamma}_{a'} [1 + A_{cc'}] \Gamma_b \Gamma_{b'} \qquad D_b D_{b'} \Phi_{bb'} \\
&+ \bar{\Gamma}_a \bar{\Gamma}_{a'} [1 + A_{bb'}] \Gamma_c \Gamma_{c'} \qquad D_c D_{c'} \Phi_{cc'} \\
&+ \bar{\Gamma}_a \bar{\Gamma}_{a'} [A_{bb'} + A_{cc'}] \Gamma_a \Gamma_{a'} D_a D_{a'} \Phi_{aa'}, \qquad aa' \neq bb' \neq cc' \neq aa'
\end{aligned} \tag{24}$$

4-body equations with pair annihilation

- This can be written as a set of two equations for the two, MM and $D\bar{D}$, components of the tetraquark,

$$\begin{aligned} \Phi_D &= \Phi_{12,34} \\ \Phi_M &= \Phi_{13,24} = -\Phi_{14,23} \end{aligned} \tag{25}$$

The relation, $\Phi_{13,24} = -\Phi_{14,23}$, for the MM component of the tetraquark follows from antisymmetry of the diquark and antidiquark wave functions, $\Gamma_{12} = -\Gamma_{21}$ and $\Gamma_{34} = -\Gamma_{43}$, with respect to permutation of the quarks quantum numbers. This antisymmetry property relates $MM \leftarrow D\bar{D}$ transition kernels to each other,

$$\bar{\Gamma}_{13}\bar{\Gamma}_{24}[1 + A_{14,23}]\Gamma_{12}\Gamma_{34} = -\bar{\Gamma}_{14}\bar{\Gamma}_{23}[1 + A_{13,24}]\Gamma_{12}\Gamma_{34} \tag{26}$$

which in turn leads to the relation, $\Phi_{13,24} = -\Phi_{14,23}$, for the solution of the Eq. (24).

4-body equations with pair annihilation

- The two above mentioned equations for Φ_M and Φ_D consist of two lines of the Eq. (24), one corresponding to $aa' = 13, 24$ ($bb' = 12, 34$, $cc' = 14, 23$, then $A_{bb'} = 0$), another corresponding to $aa' = 12, 34$ ($bb' = 13, 24$, $cc' = 14, 23$):

$$\begin{aligned} \Phi_M &= \left(\bar{\Gamma}_{13}\bar{\Gamma}_{24}A_{14,23}\Gamma_{13}\Gamma_{24} - \bar{\Gamma}_{13}\bar{\Gamma}_{24}\Gamma_{14}\Gamma_{23}\right) MM\Phi_M \\ &+ \bar{\Gamma}_{13}\bar{\Gamma}_{24}[1 + A_{14,23}]\Gamma_{12}\Gamma_{34}D\bar{D}\Phi_D, \\ \Phi_D &= 2\bar{\Gamma}_{12}\bar{\Gamma}_{34}[1 + A_{14,23}]\Gamma_{13}\Gamma_{24}MM\Phi_M \\ &+ 2\bar{\Gamma}_{12}\bar{\Gamma}_{34}A_{13,24}\Gamma_{12}\Gamma_{34}D\bar{D}\Phi_D \end{aligned} \tag{27}$$

where we have used, $A_{14,23} = A_{14} + A_{23}$, and that analogously to Eq. (26)

$$\bar{\Gamma}_{12}\bar{\Gamma}_{34}[1+A_{14,23}]\Gamma_{13}\Gamma_{24}-\bar{\Gamma}_{12}\bar{\Gamma}_{34}[1+A_{13,24}]\Gamma_{14}\Gamma_{23} = 2\bar{\Gamma}_{12}\bar{\Gamma}_{34}[1+A_{14,23}]\Gamma_{13}\Gamma_{24} \tag{28}$$

$$\bar{\Gamma}_{12}\bar{\Gamma}_{34}[A_{13,24} + A_{14,23}]\Gamma_{12}\Gamma_{34} = 2\bar{\Gamma}_{12}\bar{\Gamma}_{34}A_{13,24}\Gamma_{12}\Gamma_{34}. \tag{29}$$

Discussion

- Eqs(27) reduce to ones from (**?**) if we set $V^S_{q\bar{q}} = 0$ which is equivalent to $A_{aa'} = 0$ in Eqs(27). Kernels involving $A_{aa'}$ correspond to quark box diagrams where two-body $q\bar{q}$ intermediate states are accounted for. In this way the two-body $q\bar{q}$ component contributions are buried in the kernels of the Eqs(27) which is written in terms of only meson and diquark degrees of freedom.
  Adding these box diagrams in a sense does not complicate tetraquark equations of (**?**) because they are one loop diagrams as the kernels in (**?**) are. The complication is that one gets two equations instead of one. It is worth mentioning that anything beyond this (one loop kernel) approximation involves two and more loops in the kernel.
  Although the box diagrams could be discarded on the basis of physics arguments it is better to see directly whether they can be neglected indeed.

Discussion

- The meson-diquark picture of a tetraquark follows from the factorization approximation for the input two-body scattering amplitudes. Adding the box diagrams is not beyond this approximation; one only adds some disconnected parts in $q\bar{q}$ channels to make equations applicable to the $2q2\bar{q}$ system. This adding restores missing topologies, it is not a part of dynamics; all dynamics is incoded in the two-body scattering amplitudes.
  As one could see the disconnected parts, A_a, in $q\bar{q}$ channels cause double-counting problems which lead to a nontrivial rearrangement of the equations. Just this nontrivial rearrangement problem is addressed in the talk.

Checking over- and under-counting

- Although the physically transparent form of the final Eq. (27) should not raise doubts of that some important parts are missing or some parts are overcounted there exists a rigorous way to make this sure. To formulate a few-body approach in the QFT for a system of particles where some of them can be absorbed by others (e.g. π by N) or pair (e.g. $q\bar{q}$) annihilation takes place (these are πNN, $2q2\bar{q}$, etc. systems) one starts with the general structure of the full few-body Green function which in the case of the $2q2\bar{q}$ system is manifested by the relation,

$$G^{(4)} = G^{(4)}_{ir} + G^{(4-2)} G^{(2)} G^{(2-4)}, \tag{30}$$

where $G^{(2)}$ is the full two-body $q\bar{q}$ Green function, $G^{(4)}_{ir}$ is the $q\bar{q}$ irreducible part of the full 4-body $2q2\bar{q}$ Green function, $G^{(2-4)}$ ($G^{(4-2)}$) is the sum of all $q\bar{q}$ irreducible diagrams of the Green function corresponding to the transition $q\bar{q} \leftarrow 2q2\bar{q}$ ($2q2\bar{q} \leftarrow q\bar{q}$). DRAWING

Checking over- and under-counting

- The main problem is to express $G^{(2-4)}$ and $G^{(4-2)}$ in terms of $G^{(4)}_{ir}$. To be consistent with the problem setting (which is in the exposition of the $2q2\bar{q}$ intermediate states) $G^{(2)}$ also should be expressed in terms of $G^{(4)}_{ir}$ (this is the way to expose the $2q2\bar{q}$ intermediate states in $G^{(2)}$). This is the way we approached the πNN problem in (**?**).
  Using some algebra we get the representation (30) for the particular case of our Eq. (27).
  where N is the single quark exchange $MM \leftarrow q\bar{q}$ transition amplitude, analogously $\bar{N}$ is $q\bar{q} \leftarrow MM$ transition amplitude. Then according (34, 35) we get

$$G = G_{ir} + G_{ir} N G_{q\bar{q}} \bar{N} G_{ir}. \tag{31}$$

  where $G_{q\bar{q}}$ is the $q\bar{q}$ Green function which contains all $q\bar{q}$ intermediate states, it satisfies the following equation

$$G_{q\bar{q}} = G_0^{q\bar{q}} + G_0^{q\bar{q}}[\bar{N} G_{ir} N] G_{q\bar{q}} \tag{32}$$

Checking over- and under-counting

- Here $\bar{N}G_{ir}N$ is the $q\bar{q}$ interaction potential. In Eq. (31) the $q\bar{q}$ cuts, $G_0^{q\bar{q}}$, are exposed via, $G_{q\bar{q}}$, and the Eq. (32) for it. The Eq. (31) is the Eq. (30) in the particular case of the approximate Eq. (**??**). It is interesting that in this approximate case we have not come across with the problem of ambiguity of the very last $2q2\bar{q}$ cut in $G^{(2-4)}$ mentioned below Eq. (30). Careful analysis of this model may lead us to the solution of this well known problem in the general case. The thing is that the very last $2q2\bar{q}$ cut is not unique even in $\bar{N}G_{ir}$ (after we re-express the meson and diquark degrees of freedom in terms of the quark ones via replacement, $\Gamma_a M\bar{\Gamma}_a \to T_a$), which is the sum of a part of the diagrams involved in $G^{(2-4)}$ but nevertheless $\bar{N}$ is determined unambiguously in Eq. (31). This fact teaches us that a criterion may be worked out which helps one to make an unambiguous choice out of a few possible very last cuts such that a double-counting problem is avoided.
  The decomposition (31) allows one to see whether something is overcounted and what is missing in the initial approximate Eq. (**??**) and how can it be improved.

Derivation of (31)

- The inhomogenious equation for the MM Green function, G, corresponding to the homogenious Eq. (**??**) is

$$G = G_0^M + G_0^M (V^S_{q\bar{q}} + V^S_{2q2\bar{q}})G. \tag{33}$$

It can be written in the form

$$G = G_{ir} + G_{ir} V^S_{q\bar{q}} G. \tag{34}$$

where G_{ir} the sum of all $q\bar{q}$ irreducible terms in the Green function, G, it satisfies the equation

$$G_{ir} = G_0^M + G_0^M V^S_{2q2\bar{q}} G_{ir}. \tag{35}$$

$$\text{where} \qquad V_{q\bar{q}} = N G_0^{q\bar{q}} \bar{N}, \tag{36}$$

$G_0^{q\bar{q}} = G^0_{14}$ is the two-body propagator of non-interacting q and $\bar{q}$, N is the single quark exchange $MM \leftarrow q\bar{q}$ transition amplitude, analogously $\bar{N}$ is $q\bar{q} \leftarrow MM$ transition amplitude.