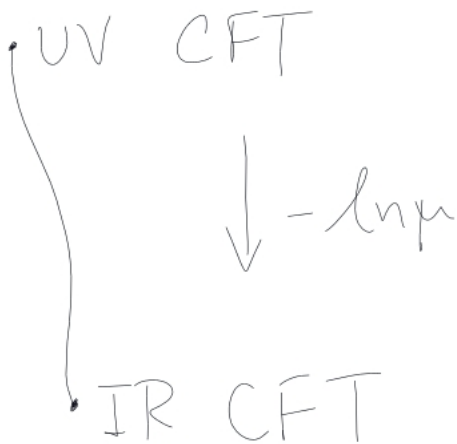


Gradient Properties of RG Flows

arXiv: 2402.17817

with William Pannell

Relativistic
Unitary
Perturbative
QFT





$$\beta^I = \hbar \frac{d\lambda^I}{d\mu} = Q^I_J \lambda^J$$

anti-symm

Monotonicity theorems \leftarrow Riemannian

• Strongest: $\beta^I = G^{IJ} \partial_J A$

• Strong: $\mu \frac{dA}{d\mu} > 0$

• Weak: $A_{UV} > A_{\mathbb{R}}$

$$\mathcal{L} = \frac{1}{2} \partial_\mu \phi_i \partial^\mu \phi^i + \frac{1}{4!} \lambda_{ijkl} \phi_i \phi_j \phi_k \phi_l$$

$$\beta_{\underbrace{ijkl}_I} = \mu \frac{d\lambda_{ijkl}}{d\mu}$$

We want to see if

$$\beta_I = G_{IJ} \partial_J A$$

If G_{IJ} is symm., then at any point:

$$G_{IJ} = \delta_{IJ}$$

$$\beta_I = \partial_I A \Rightarrow \partial_I \beta_J = \partial_I \partial_J A$$

One loop

$$\beta_{ijkl} = \lambda_{ijmn} \lambda_{mnkl} + \text{perms}$$



$$A = \lambda_{ijkl} \lambda_{klmn} \lambda_{mnij} =$$



$$G = \delta_{ijkl; mnqp} =$$

Two loops

$$z_{\beta} = z_b \text{ (diagram)} + z_e \text{ (diagram)}$$

The diagram for z_b shows a vertex with two lines extending from it, and a loop structure consisting of two vertices connected by two curved lines. The diagram for z_e shows a vertex with three lines extending from it, and a loop structure consisting of two vertices connected by two curved lines.

$$z_A = z_{a_1} \text{ (diagram)} + z_{a_2} \text{ (diagram)}$$

The diagram for z_{a_1} shows a circle with two vertices on its circumference, connected by two curved lines. The diagram for z_{a_2} shows a circle with four vertices on its circumference, connected by four curved lines forming a diamond shape.

$$+ z_{a_3} \text{ (diagram)}$$

The diagram for z_{a_3} shows a circle with four vertices on its circumference, connected by four curved lines forming a sphere-like structure.

$$z_G = z_g \text{ (diagram)}$$

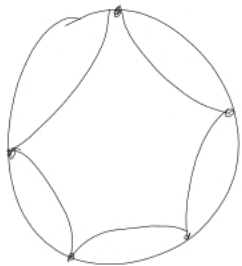
The diagram for z_g shows a vertex with two lines extending from it, and a loop structure consisting of two vertices connected by two curved lines.

	1	2	3	4	5	6
β	1	2	7	23	110	571
A	1	3	5	17	42	177
G	1	1	7	18	97	453
Eqs.	1	3	10	36	164	819

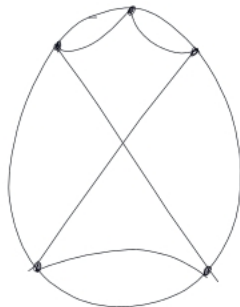
Three loops



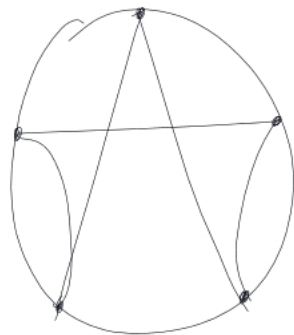
3



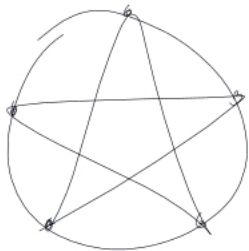
1



3



2

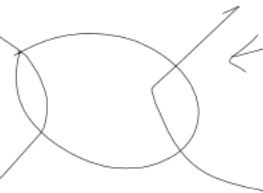


1

~



$\frac{1}{6}$



$\frac{1}{3}$



$$3 \int_{b_1} = 2 \int_{b_2}$$

" 1

$-\frac{1}{2}$

" 2
2

At 3 loops metric cannot be globally flat.

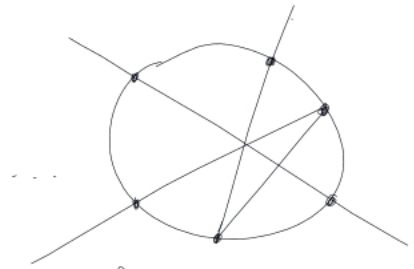
Four loops and beyond

4 : 4 equations ✓

5 : 37 equations ✓

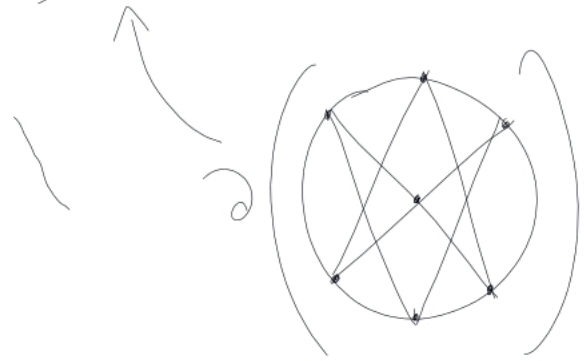
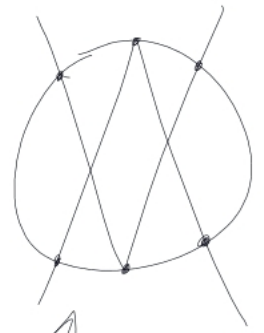
β_5

\cup



+

...



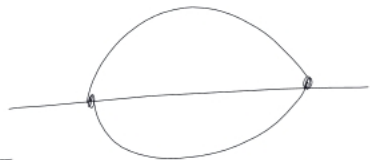
$36 \beta_3^2$

6 loops : 234 $\begin{matrix} \nearrow 229 \checkmark \\ \searrow 5 \times \end{matrix}$

Options: (A) Someone is wrong
(B) RG flow is gradient
(C) None of the above

In dim-reg

$$\phi_B = z^{1/2} \phi_R$$



$$(z^{1/2})^T z^{1/2} = (z^{1/2})^T \underbrace{0^T 0}_1 z^{1/2} = (0z^{1/2})^T (0z^{1/2})$$

Key: $T^\mu{}_\mu = \beta^i \mathcal{O}_i + \partial^\mu J_\mu$

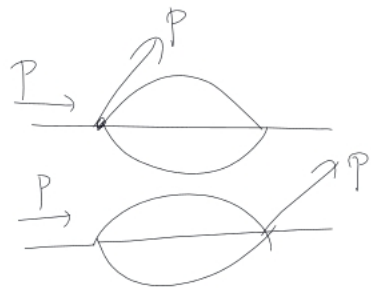
$$J_\mu = S_{ij} \phi_i \partial_\mu \phi_j$$

$$S_{ij} = -S_{ji}$$

$$T^\mu{}_\mu = \mathcal{B}^i \mathcal{O}_i, \quad \mathcal{B}^i = \beta^i - (S\lambda)^i$$

$$(N_I)_{ij} \phi_i \partial_n \lambda^I \partial^n \phi_j$$

$$(N_I)_{ij} = \sum_n \frac{(N_I^{(n)})_{ij}}{\varepsilon^n}$$



$$S_{ij} = (N_I^{(1)})_{ij} \lambda^I$$



$${}^5S_2 = \frac{259}{4608}$$

$$192 {}^5S_3 + 384 {}^5S_4 = 31-36J_3$$

5 loops : 4 S-diagrams

6 loops : 19 S-diagrams

$$B = \beta - \gamma \lambda$$

$$\beta \rightarrow \beta + w \lambda$$

$$\gamma \rightarrow \gamma + w$$

$$\gamma \rightarrow \gamma + w$$

$$\Gamma = \gamma + S$$

$$\beta \rightarrow \beta - \omega \lambda$$

$$S \rightarrow S + \omega$$

$$J_{\mu} = S_{i0} \phi_i \partial_{\mu} \phi_i$$

$$B = \beta - (S\lambda)$$

$$\langle T_{\mu}^{\mu} \mathcal{O}_{\downarrow} \rangle$$

$$\langle T_{\mu}^{\mu} \partial^{\mu} J_{\nu} \rangle$$

$$(N_{\downarrow})_{i0} \partial_{\mu} \lambda^{i(x)} \phi_i \partial_{\mu} \phi_0$$