

DAVID POLAND

"BOOTSTRAPPING THE  
ISING STRESS TENSOR"

WIP w/ Chang, Erramilli, Homrich  
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# Motivation

$$\text{3d Ising CFT: } \sigma \times \sigma \sim \mathbb{1} + \Sigma + \overline{T_{\mu\nu}} + \dots$$
$$\sigma \times \Sigma \sim \sigma + \sigma' + \dots$$

Numerics w/

- $\langle \sigma \sigma \sigma \sigma \rangle$
- $\langle \sigma \sigma \Sigma \Sigma \rangle$
- $\langle \Sigma \Sigma \Sigma \Sigma \rangle$



$$\Delta \sigma' > 3$$

$$\Delta \Sigma' > 3$$

$$\text{OPE: } \frac{\lambda_{\sigma\sigma\Sigma}}{\lambda_{\Sigma\Sigma\Sigma}}$$

$$\Delta_{\sigma} = 0.5181489(10)$$

$$\Delta_{\varepsilon} = 1.412625(10)$$

$$\lambda_{\sigma\varepsilon\varepsilon} = 1.0518537(41)$$

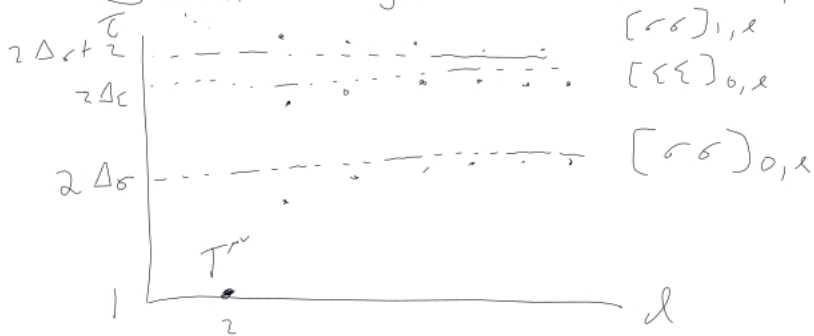
$$\lambda_{\varepsilon\varepsilon\varepsilon} = 1.5132435(19)$$

Missing several things:

- No parity odd

-  $Z_2$ -even, odd spin

- Several trajectories in extremal spectra



No clear sign of  $[\nabla T]$ ,  $[\varepsilon T]$ ,  $[\sigma T]$ , ...

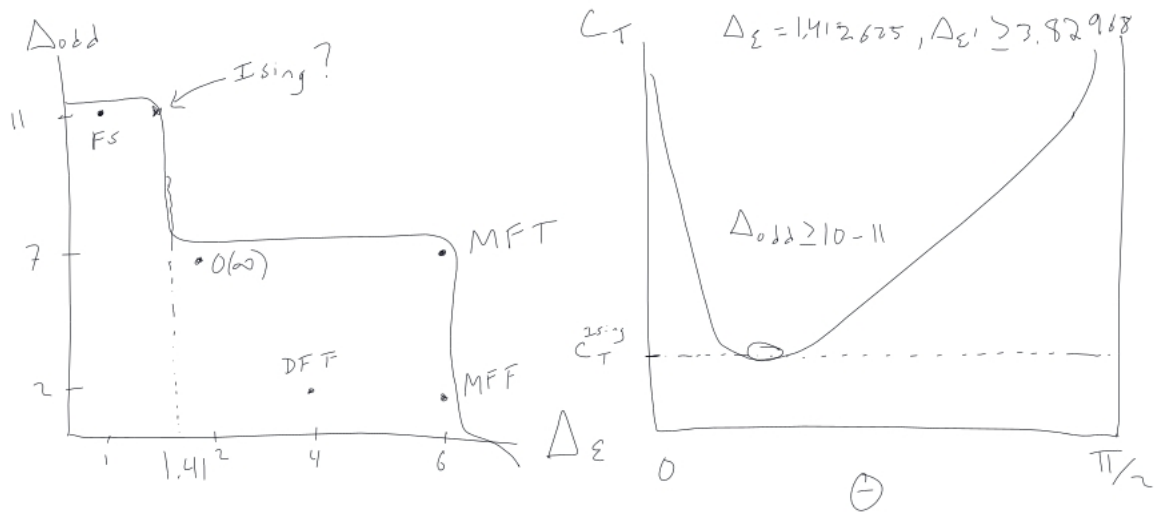
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Separately, progress in  $\langle TTTT \rangle$

$$T \times T \sim \mathbb{1} + \lambda_{TT\varepsilon} \varepsilon + \lambda_{TTT} T + \lambda_{TT\Theta} \Theta_{\text{odd}} + \dots$$

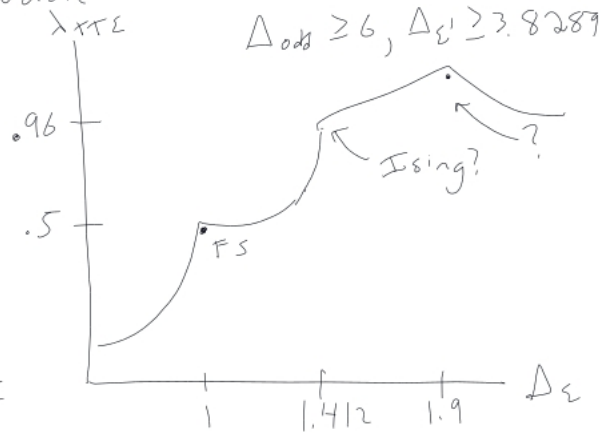
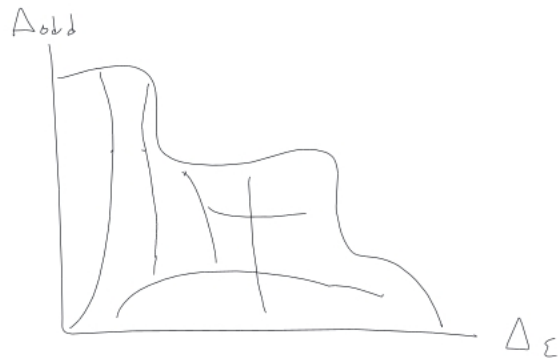
$$\langle TTT \rangle = n_B \langle TTT \rangle_B + n_F \langle TTT \rangle_F$$

$$\text{Ward: } n_B + n_F = C_T \quad \left| \quad \text{Param: } n_B = C_T \frac{\cos \theta}{\cos \theta + \sin \theta} \right.$$
$$n_F = C_T \frac{\sin \theta}{\cos \theta + \sin \theta}$$



$$\Rightarrow 0.01 \leq \theta \leq 0.019$$

$\lambda_{TTE}$  bounds have lots of structure



Ising kink near  $\lambda_{TTE} \sim .96$

SPT bootstrap :  $\lambda_{TTE} \sim .958(7)$

Fuzzy sphere :  $\lambda_{TTE} \sim .916(7)$

Combine  $\{T, \sigma, \varepsilon\}$

- Many tensor structures, BUT Few indep. eqs.

due to conservation  
2-variable ( $z, \bar{z}$ )

$\partial_r T^r = 0$

1-variable ( $z = \bar{z}$ )

Const ( $z = \bar{z} = 1/2$ )

$$\langle TTTT \rangle \Rightarrow 5$$

9

4

$$\langle TTT\varepsilon \rangle \Rightarrow 4$$

3

1

$$\langle TT\sigma\sigma \rangle \Rightarrow 6$$

3

0

$$\langle T\varepsilon\varepsilon\varepsilon \rangle \Rightarrow 6$$

3

0

$$\langle T\{\varepsilon\varepsilon\} \rangle \Rightarrow 1$$

0

0

$$\langle T\varepsilon\sigma\sigma \rangle \Rightarrow 3$$

0

0

$$+ \{\sigma, \varepsilon\} \Rightarrow 5$$



- Set up Crossing Ears using Haskell Codebase  
(hyperion-bootstrap, ...)

- Compute blocks using blocks\_3d

- Solve using Sdpb 3.0.0

+ new polynomial sampling method

⇒ Much better condition #'s ( $10^{150} \rightarrow 10-100$ )

Search through 6d parameter space

$$\left\{ \Delta_{\sigma}, \Delta_{\varepsilon}, \frac{\lambda_{TTTT}^B}{\lambda_{TTTE}}, \frac{\lambda_{TTTT}^F}{\lambda_{TTTE}}, \frac{\lambda_{\sigma\sigma\varepsilon\varepsilon}}{\lambda_{TTTE}}, \frac{\lambda_{\varepsilon\varepsilon\varepsilon\varepsilon}}{\lambda_{TTTE}} \right\}$$

Using gff navigator + Skydive algorithm

Assumptions:  $\Delta_{\varepsilon'} \geq 3$

$$\Delta_{\sigma'} \geq 3$$

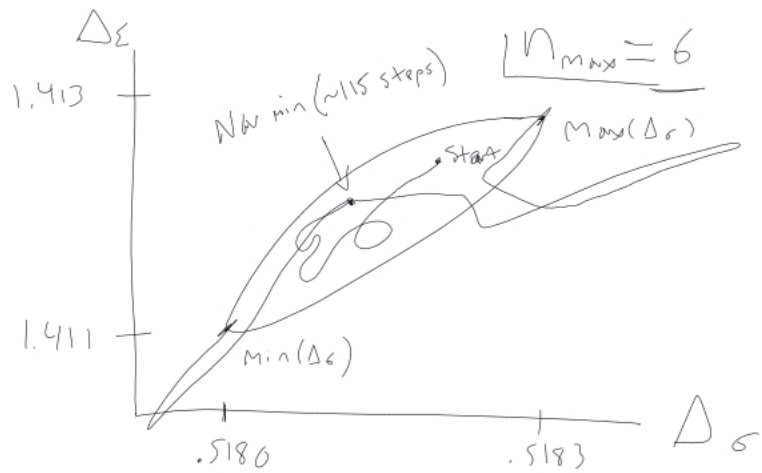
$$\Delta_{\text{odd}} \geq 3$$

$$\Delta_{T'} \geq 4$$

Add to sum rule:

$$0 = \dots + \lambda \left( 2 \overline{\overline{F}}_{2\Delta_\sigma, 0} + 2 \overline{\overline{F}}_{2\Delta_\varepsilon, 0} + \overline{\overline{F}}_{\Delta_\sigma + \Delta_\varepsilon, 0} \right. \\ \left. + \# \overline{\overline{F}}_{2\Delta_\sigma + 2, 2} + \# \overline{\overline{F}}_{2\Delta_\varepsilon + 2, 2} \right)$$

$$\mathcal{N} = \min(\lambda)$$



$\Delta \varepsilon$  $N_{max} = 14$ old  $\{\sigma, \varepsilon\}$  island  
( $N_{max} = 22$ )min( $\Delta \sigma$ )max( $\Delta \sigma$ )min( $N_{max}$ )

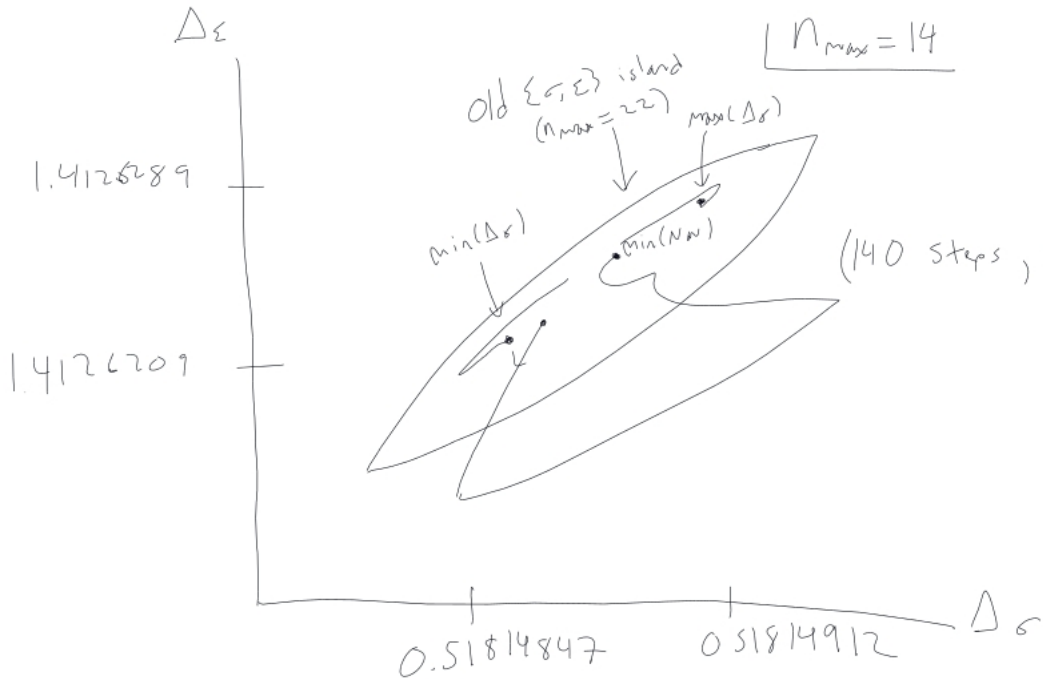
(140 steps, ~150k core-hours)

1.4128289

1.4126209

0.51814847

0.51814912

 $\Delta \sigma$ 

Using know here, get results:

$$\lambda_{TTE} = 0.953315(11)$$

$$n_B = 0.933445(11)$$

$$n_F = 0.01309425(59)$$

$$\Rightarrow C_T = 0.946538(11)$$

$$\Theta = 0.01402696(58)$$

## Future

- Finish Skydiver at  $N_{\max} = 18$
- Extremal Spectrum, Study  $[TT]$ ,  $[TS]$   
+ trajectories
- Determine  $\Delta_{\text{odd}}$   
( $N_{\max} = 10: \Delta_{\text{odd}} < 10.954$ )
- Explore other structures in  $\{T, S\}$