IMPURITIES WITH A CUSP

based on 2406.10186 w. Y.C.-He & Z. Komargodski

(see also · O. Dyatlov, M. Khanchandani, F. Popov, Y. Wang
2404.05815 + 2406.01550
· P. Kravchuk, A. Radcliffe, R. Sinha 2406.01550)

Plan

1) DCFT review

2) Cusp anomalous dimensions

3) Results for the localized magnetic field

DCFT review

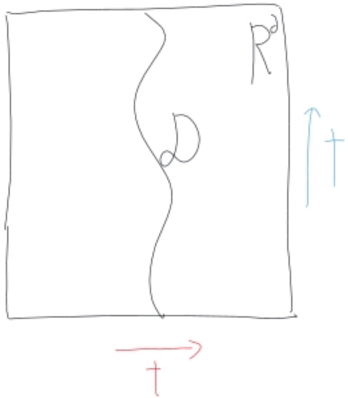
2 viewpts on defect

1) $\mathcal{D}$ = non-local operator

2) QFT + $\mathcal{D}$ = Defect QFT

$$H = H_{bulk} + H_{imp}$$

$$H_{imp} \sim \delta^{d-1}(\vec{x})\,\hat{O}$$



· Defect CFTs = DQFTs invariant under

$$\color{skyblue}{SL(2,\mathbb{R})} \times \color{red}{SO(d-1)} \subset SO(d+1,1)$$

- defect ops ≠ bulk ops:

$$\langle \hat{O}(\tau)\,\hat{O}(0)\rangle_{\mathcal{D}} = \frac{1}{\tau^{2\hat{\Delta}}}$$

- bulk to defect OPE

$$O(\tau, x_\perp) \sim \sum_{\hat{O}} b_{O\hat{O}}\, |x_\perp|^{\hat{\Delta}_{\hat{O}}-\Delta_O}\, \hat{O}(\tau)$$

- Displacement op:

$$\partial_\mu T^{\mu\nu} = -\hat{D}^\nu \delta^{d-1}(x_\perp)$$

$$\log\left(\frac{Z_{DCFT}(X^\mu+\delta X^\mu)}{Z_{DCFT}(X^\mu)}\right) = \int d\tau\, \delta X^\nu \langle \hat{D}_\nu \rangle$$

$\mathcal{D}$

$X^\mu(\tau)$

$\delta X^\mu(\tau)$

• Massive DQFT flow under RG:

$$S_{DQFT} = S_{DCFT_{UV}} + M^{1-\hat{\Delta}} \int d\tau\, \hat{O}_{\hat{\Delta}} \quad , \hat{\Delta} < 1$$

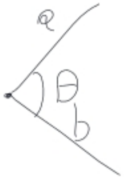
• Ex: localized magnetic field

runs

$$D_h = e^{h \int_{\vec{x}=0} d\tau\, \hat{\sigma}(\tau)}$$

$$H_{imp} = h\, \delta^{d}(\vec{x})\, \hat{\sigma}$$

Cusp anomalous dimension

- Cusp  $\simeq$ local operator

$$\left(a \to e^{\int d\tau M} a\right)$$

$$\log Z_{ab}/Z_{CFT} = -\Gamma_{ab}(\theta)\log\left(\frac{L_{IR}}{a_{UV}}\right)$$

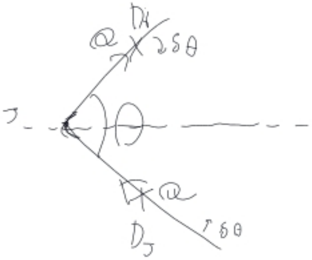
cusp anomalous dimension

- State-op map:

$$\Gamma_{ab}(\theta) = E_{ab}(\theta) - \frac{1}{2}E_{aa}(\pi) - \frac{1}{2}E_{bb}(\pi)$$

• Concavity:

$$\frac{\partial^2 \log Z_{a\bar a}(\theta)}{\partial\theta^2} = \int_0^\infty d\tau_1 \int_{-\infty}^0 d\tau_2 \, \underbrace{\langle D_I(\tau_1) D_J(\tau_2)\rangle_c}_{\frac{1}{\tau_1^2}\frac{1}{\tau_2^2}F(\tau_1/\tau_2)} \, \partial_\theta X^i \partial_\theta X^j$$

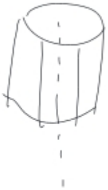


$\langle 0| D_a D |0\rangle_c$

$\sum_n |n\rangle\langle n|$

$$= \log\left(\frac{L}{a}\right) \int_0^\infty \frac{dx}{x} F(x) > 0$$

$\langle \hat O D D \hat O \rangle$



- convergence:

wrt to defect OPE $\longrightarrow$ $F(x) = \begin{cases} x^{\Delta_{min}} & x \to 0 \\ x^{-\Delta_{min}} & x \to \infty \end{cases}$

$\longrightarrow$ $\Gamma''_{a\bar a}(\theta) < 0$

- $\theta \to \pi$ limit:

$$\Gamma_{ab}(\theta) = \Delta_{ab} + \frac{1}{2}\beta\,(\pi-\theta)^2 + \dots$$

$$\langle D\hat{O}_{ab}\, D\hat{O}_{ba}\rangle \xrightarrow{b=a} \langle D_i D_j\rangle = \frac{C_D\,\delta_{ij}}{\tau^4}$$

$$\beta = -\frac{C_D}{6}$$

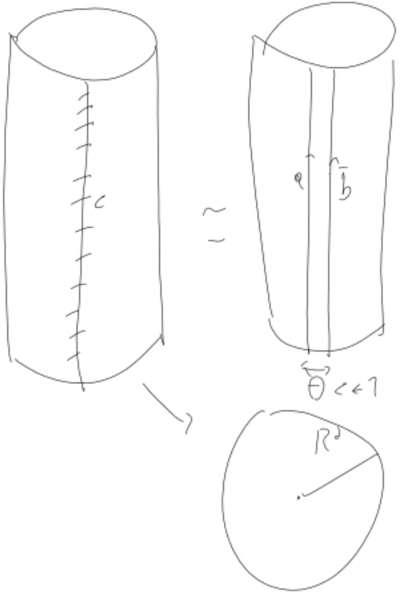
- $\Theta \to 0$: fusion

$$a(\Theta)\bar{b}(0) = \bigoplus_c e^{-\int d\tau \frac{C_{a\bar{b}c}}{\Theta}} c(0)\, e^{\sum_{\text{irr}} \overbrace{\int d\tau\, O_{\text{irr}}}^{\Theta^{\Delta_{\text{irr}}-1}}}$$

smaller Casimir energy
is retained

$$\simeq e^{-\int d\tau \frac{C_{a\bar{b}c}}{\Theta}} c(0)\, e^{\int \text{irr. pert.}}$$

$$\Gamma_{ab}(\Theta) = \frac{C_{a\bar{b}c}}{\Theta} + \Delta_{c0} + \#\,\Theta^{\Delta_{\text{irr}}-1} + \dots$$

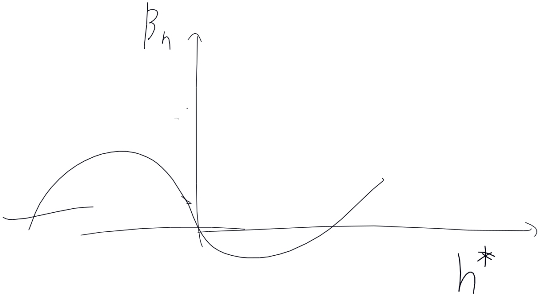


$b = a \longrightarrow C_{a\bar{a}1} < 0$
[Io & Bachas]

Localized magnetic field defect

Ising $\mathcal{D}_h = e^{h\int dz\, \hat{\sigma}}$

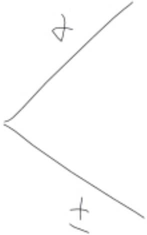
$d = 4-\epsilon$:



$\beta_h = -\frac{\epsilon}{2}h + \frac{\lambda}{6(4\pi)^2}h^3$

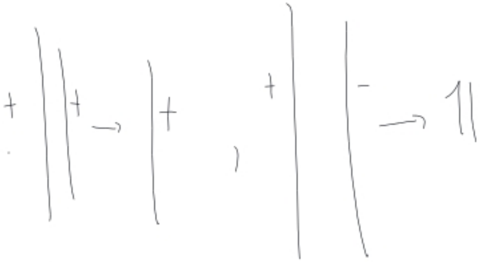
$\Delta_{\hat{\varphi}} = 1+\epsilon+\dots > 1$

$$\Gamma_{\pm\alpha} = \mp \frac{h_*^2}{4\pi^2}\frac{\pi-\theta}{\sin\theta} + \frac{h_*^2}{4\pi^2} + O(\varepsilon) \quad , \quad h_* = \sqrt{g}$$



$$\to \Gamma_{+-} \overset{\theta\to 0}{=} \frac{C_{+-0}}{\theta} + 0 + O(\theta)$$

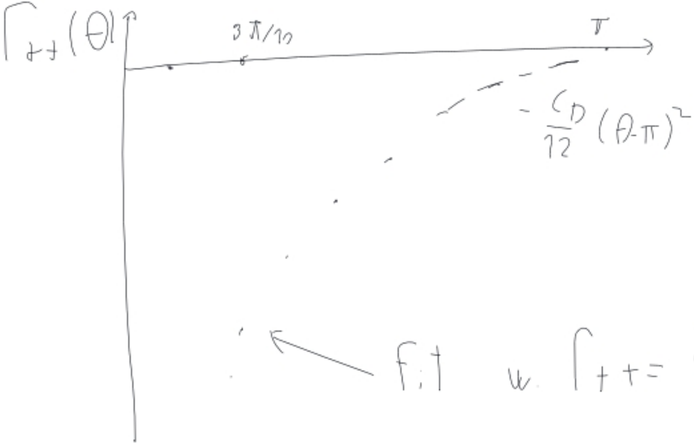
$$\to \Gamma_{++} \overset{\theta\to 0}{=} \frac{C_{+++}}{\theta} + \text{const.} + \varepsilon\,\#\log\theta + \dots$$

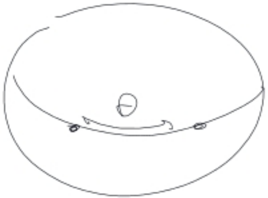


$$\xrightarrow{\text{Callan-Sym.}} \frac{C_{+++}}{\theta} + \frac{\Delta_{+0}}{1+\#\theta^{\varepsilon}} + \dots \qquad \Delta_{\hat{\phi}} = 1+\varepsilon$$

$$S_{\text{drr}} = \theta^{\varepsilon}\int d\tau\, \hat{o}$$

$d=3$ Ising w. Fuzzy sphere





measured before
$\simeq 0.12$

fit w. $\Gamma_{++} = \frac{C_{+++}}{\theta} + \Delta_{rot}\,\alpha\,\theta^{\Delta_{\hat\sigma}-1} + \dots$

$\Delta_{\hat\sigma} \simeq 1.6$