Last time:

→ QFTs naturally have: - Hilbert space $\mathcal{H}$

- vN algebras $\mathcal{A}(\mathcal{O})$ for subregions.

→ In (non-gauge) lattice theories subsystem physics is encoded in density ops ρ_A.

↳ problems in continuum & in gauge theories

Sometimes: Introduce ϵ.

Sometimes: Other tools.

Idea: $\rho_A \otimes 1_B$, $1_A \otimes \rho_B$, individually UV-divergent

Maybe $\underbrace{\rho_A \otimes \rho_B^{-1}}_{\Delta_\Psi \text{ modular operator}}$ is fine?

$$-\log \Delta_\Psi \sim \underset{-\log \rho_A}{K_A} - K_B \equiv K_\Psi$$

"full" modular Hamiltonian

"Tomita-Takesaki" (see 2309.16766)

Def: $|\Psi\rangle \in \mathcal{H}$, A vNA

$|\Psi\rangle$ is cyclic w/r/t A if

$$\{a|\Psi\rangle \mid a \in A\} \text{ is dense in } \mathcal{H}. \Bigg\} \begin{array}{l} \rho_{A^c} \\ \text{full-rank} \end{array}$$

Def: $|\Psi\rangle$ is fully entangled for A if

it is cyclic for A and for A'

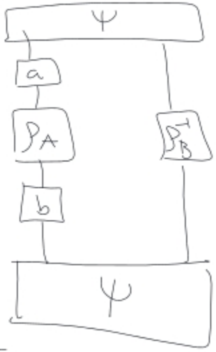
"cyclic and separating"

<u>Thm:</u> (Tomita)

$\mathcal{H}$ Hilb.

A vNA

$|\psi\rangle$ fully entangled



$|\psi\rangle = \sum \sqrt{p_i}\, |i\rangle_A |i\rangle_B$

$\rho_A = \sum p_i\, |i\rangle_A \langle i|_A$

$\exists \; \Delta_\psi \geq 0$

- $a, b \in A$

$$\Rightarrow \langle \Delta_\psi^{1/2} a \psi | \Delta^{1/2} b \psi \rangle = \langle b^\dagger \psi | a^\dagger \psi \rangle$$

$$\text{"} \langle \psi | a^\dagger \Delta_\psi b | \psi \rangle = \langle \psi | b a^\dagger | \psi \rangle \text{"}$$

$\rho_A^{(\psi)} \otimes \rho_B^{-1(\psi)}$

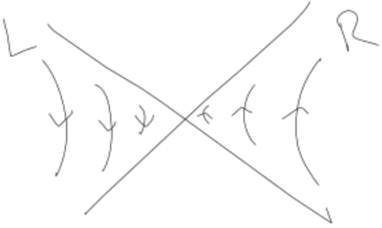
- $K_\psi = -\log \Delta_\psi$ exists
- $e^{iK_\psi t} A e^{-iK_\psi t} = A$ ← modular flow

Example: The Unruh effect

Heuristic: "The Minkowski $|\Omega\rangle$ looks thermal when probed by a boost."

1976 (Unruh)

1983



Precise: The modular flow of $|\Omega\rangle$ in R is a boost.

$K_\psi = -\log \rho_R + \log \rho_L$

(Bisognano–Wichmann) 1975

Could be true — $|\Omega\rangle$ is cyclic for every region (Reeh–Schlieder)

$\Rightarrow$ Mod. flow is defined

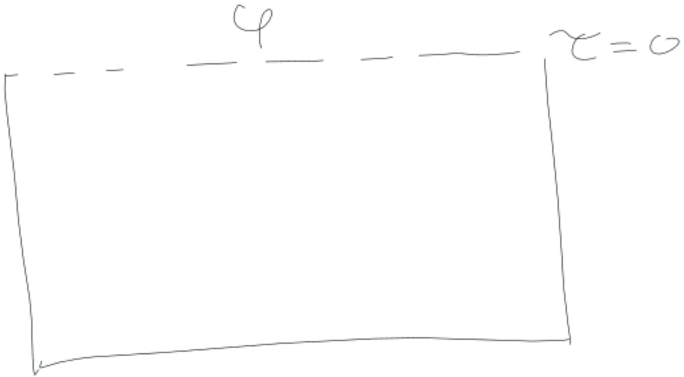
?

Philosophy

Compute well defined things in heuristic ways.

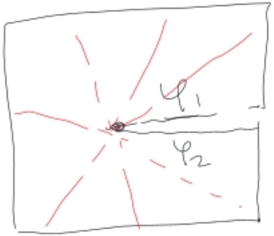
Compute $-\log \rho_R + \log \rho_L$

$\langle \varphi | \Omega \rangle \sim$



$\rho_R = \mathrm{tr}_L |\Omega\rangle\langle\Omega|$

$$\langle \varphi_2 | \rho_R | \varphi_1 \rangle \sim$$



$$\rho_R \sim T\exp\left[-\int_0^{2\pi} d\theta \;\; K_\theta \right]$$

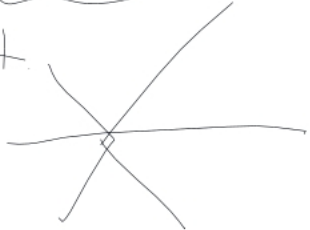
$$K_\theta = \int_{\text{const.}\,\theta} T^E_{\mu\nu} \left(x\partial_\tau^\mu + \tau\partial_x^\mu \right) d\Sigma^\nu$$

$$\sim \exp\left[-2\pi \int_0^\infty dx\, dy\; x\, T^E_{\tau\tau} \right]$$

$$\rho_R = \frac{1}{Z} \exp\left[-2\pi \int_0^\infty dx \int d\vec{y} \; x \, T_{00} \right]$$

$$\rho_L = \cdots \exp\left[+ \int_{-\infty}^{0} \right]$$

$$K_R = -\log \rho_R + \log \rho_L = 2\pi \underbrace{\int_{-\infty}^{\infty} dx \, d\vec{y} \; x \, T_{00}(t=0, x, \vec{y})}_{\text{Boost}}$$



<u>Thm</u>: $\mathcal{A}$, $|\Psi\rangle$, A $\qquad \Delta_\Psi$, K_Ψ

- $e^{iK_\Psi t} A e^{-iK_\Psi t} = A$
- $e^{iK_\Psi t}|\Psi\rangle = |\Psi\rangle$

K_Ψ is the <u>unique</u> op satisfying this

- $F_{ab}(s) = \langle\Psi| e^{iK_\Psi s} a e^{-iK_\Psi s} b |\Psi\rangle$

$a, b \in A$

$F_{ab}(z)$

$F_{ab}(s)$

$z = s$

$\langle\Psi| b e^{iK_\Psi s} a e^{-iK_\Psi s} |\Psi\rangle$

$z = s - i$

ANEC

History:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi T_{\mu\nu}$$

Constrain $T_{\mu\nu} \rightarrow$ constrain g

Classically

$$K^\mu \text{ null} \Rightarrow T_{\mu\nu} K^\mu K^\nu \geq 0 \quad \text{(NEC)}$$

Prove:

- → Sing. theorems
- → Area theorem
- → ...

Problem: No NEC in QFT $\left(\langle T_{kk} \rangle < 0 \right)$

Good: Evaporate

Bad: no control

Some cases:
suffices

ANEC

$$\int_\gamma T_{\mu\nu} k^\mu k^\nu \geq 0$$

Not true

Good evidence in Minkowski $\leftarrow$ good for constraining QFT

1605.08072

ANEC flow and relative entropy

$$\mathrm{ANEC}[\vec{y}] = \int_{-\infty}^{\infty} dx^+ \, T_{++}(x_+, x_- = 0, \vec{y})$$

Q: Is $\langle \Psi | \mathrm{ANEC}[\vec{y}] | \Psi \rangle \geq 0$?

Idea: $$[\mathrm{ANEC}[\vec{y}_1], \phi(x_+, x_- = 0, \vec{y}_2)]$$

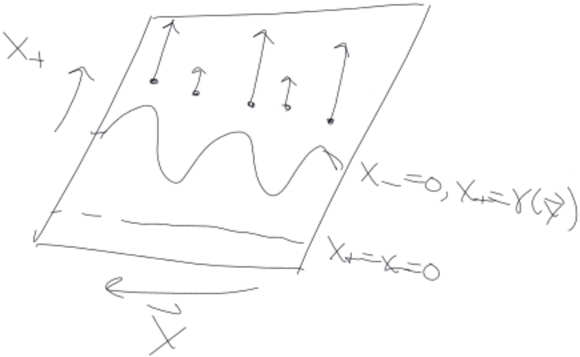
$$\sim \delta(\vec{y}_1 - \vec{y}_2) \, \partial_{x_+} \phi$$

Q: Is this flow positively generated?

Smearing: $\gamma(\vec{y})$

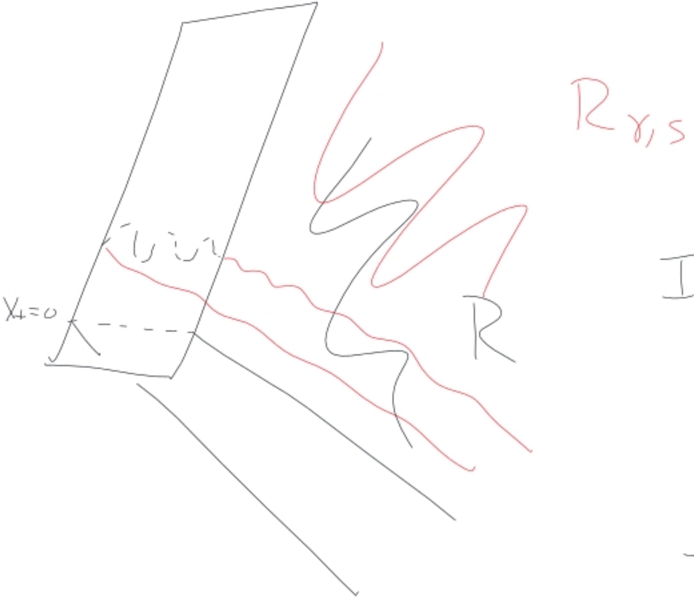
$$\mathrm{ANEC}[\gamma] = \int T_{++}\,\gamma \quad dx_+\,d\vec{y}$$

$$[\mathrm{ANEC}[\gamma], \phi(x_+, x_-=0, \vec{y})] = \gamma(\vec{y})\,\partial_{x_+}$$



Q: Is $\mathrm{ANEC}[\gamma] \geq 0$ for $\gamma \geq 0$

$e^{isANEC[\gamma]}$



Map

$$A(R) \to A(R_{\gamma,s})$$

Idea: If $ANEC[\gamma] \geq 0$

$\langle\Psi|ANEC[\gamma]|\Psi\rangle$

must quantify sth.

Relative entropy

Density matrices

$$S(\rho\|\sigma) = \operatorname{tr}(\rho\log\rho) - \operatorname{tr}(\rho\log\sigma)$$

"distinguishability"

$$S(\rho\|\sigma) \geq 0$$

"monotonicity"

$$S(\rho_{AB}\|\sigma_{AB}) \geq S(\rho_A\|\sigma_A)$$

In alg. $S_A(\Psi\|\Phi) = S(\rho_A^{\Psi}\|\rho_A^{\Phi})$

$|\Psi\rangle, \; |\Omega\rangle$

$$S_{R_{\gamma,s}}(\Psi \| \Omega) \leq S_R(\Psi \| \Omega)$$

$$-\partial_s \left(S_{R_{\gamma,s}}(\Psi \| \Omega) \right) \Big|_{s=0} \geq 0$$

Is this $\langle \Psi | ANEC[\gamma] | \Psi \rangle$

$$\underbrace{\partial_s \left[S_{\mathcal{L}_{\gamma,s}}(\Psi\|\Omega) - S_{R_{\gamma,s}}(\Psi\|\Omega)\right]_{s=0}}_{=2\pi\langle\Psi|\mathrm{ANE}[\gamma]|\Psi\rangle} \geq 0$$

$$\partial_s \langle\Psi| K^{d}_{\Omega,\mathcal{L}_{\gamma,s}}|\Psi\rangle \geq 0$$

$$\langle\Psi|K_{\Omega,\mathcal{L},s}|\Psi\rangle \quad s>0$$

$$\geq$$

$$\langle\Psi|K_{\Omega,\mathcal{L}}|\Psi\rangle$$

HSMT

$$\Rightarrow \boxed{[K_s, K] \sim 2\pi K_s}$$

$$\boxed{K_s - K \geq 0}$$

$$K_s = e^{iANEC_s} K e^{-iANEC_s}$$