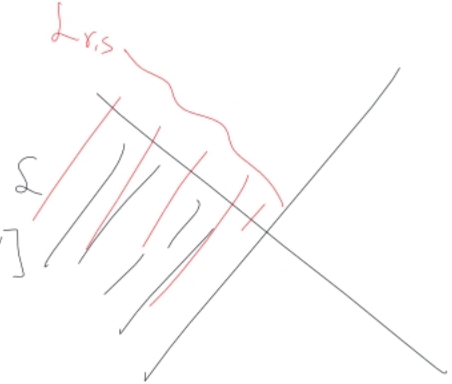
Last time

- Modular Hamiltonians $K \leftarrow (\mathcal{H}, |\Psi\rangle, A)$
- For $|\Omega\rangle$, A_{Rindler}, $K = 2\pi\,\text{Boost}$
  - Heuristic: Path integral
  - Proof: Analyticity
- For $A \subseteq B$, $K_B - K_A \geq 0$.

Suggested: $K_{\mathcal{L}_{r,s}} - K_{\mathcal{L}} = 2\pi s\, \text{ANEC}[r] + O(s^2)$

PI - see notes



This time: Black holes

Black holes thermo. $\longrightarrow$ BH stat mech?

D-branes
grav. PI
Cardy asymptotics

2112.12828 ← Witten
2206.10780
2404.16098
(2302.01958)
(2306.01837)

Matter	Grav
Thermo.	✓
C. Stat. Mech.	?
Q. Stat. Mech.	✓ in some cases

Q: Does BH thermo admit a "state-counting" description semiclassically?

→ UV-independent

→ informs QG

Idea: Is BH entropy a quantum entropy of a semiclassical alg.?

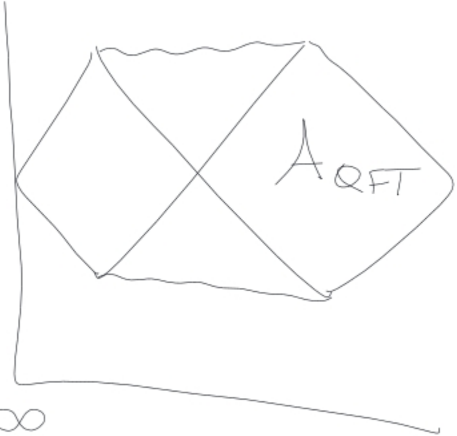
Traces & types

Witten: Forget interpretation

Do BHs have

$$-\mathrm{tr}(\rho \log \rho)\ ?$$

$\rightarrow$ Need ρ, need tr.



Typically $\rho \in A_{QFT}$ $\quad \mathrm{tr}(\rho) = \infty$

$$\left.\begin{array}{l}\text{Q: Does there exist a "ren. tr" } \tau \sim \mathrm{tr}_{\mathcal{H}} \\ \text{with } \tau(\rho) < \infty\text{? Is it unique?}\end{array}\right\} \text{type classification}$$

Def: Given $\mathcal{H}$, A, a <u>trace</u> is a map

$$\tau : A_+ \longrightarrow [0,\infty]$$

- $\tau(\lambda(a+b)) = \lambda\tau(a) + \lambda\tau(b)$ $\qquad \lambda \geq 0,\ a,b \in A_+$
- $\tau(UaU^+) = \tau(a)$ $\qquad a \in A_+,\ \underset{\text{unitary}}{U} \in A$

or

$$\tau(a^+a) = \tau(aa^+) \qquad a \in A$$

"trace-class"

Prop: τ extends uniquely to $A_1 = \{ a \in A \mid \tau(\underbrace{\sqrt{a^+a}}_{|a|}) < \infty \}$

→ linear, cyclic

<u>Prop:</u> A_1 is an ideal. $a \in A$, $\rho \in A_1$,

$a\rho, \rho a \in A_1$.

$$\Rightarrow \tau(\rho) < \infty \Rightarrow \tau(\rho a) < \infty.$$

Def: τ is a ren. tr. if

- $\tau(a) = 0$ for $a \geq 0 \implies a = 0$

  (Faithful)

- $0 \leq a_1 \leq \ldots \leq a$ $\quad a = \sup a_n$

  $\tau(a_n) \to \tau(a)$. (Normal)

- A_1 dense in A (Semifinite)

  "ultraweak"

Def: A is a <u>factor</u> if $Z(A) = \mathbb{C}\mathbb{1} \iff A \cap A' = \mathbb{C}\mathbb{1}$

Fact: All vNA's decompose as $\oplus$ factors.

Thm: On a factor, ren. tr. τ_1, τ_2

$$\tau_1 = c\,\tau_2$$

$$-\tau(\rho\log\rho) \xrightarrow{\tau \to c\tau} -\tau(\rho\log\rho) + \log c$$

$$\boxed{\tau(\rho) = 1}$$

Non-commutative L^p spaces.

Def. A factor is

no entropy

- Type $\underline{III}$: no ren. τ.

norm.
$\tau(\text{min. proj}) = 1$
absolute entropy

- Type $\underline{I}$: $\exists\ \tau$, $\tau(\text{proj.}) \geq \# \gneq 0$.

"pure states"

- Type $\underline{II}$: $\exists\ \tau$, $\tau(\text{proj.}) \to 0$

"gapless"

ΔEntropy is defined.

Crossed product

Given $\mathcal{H}, A, |\Psi\rangle \longrightarrow K$ $\swarrow$ $\hat{x}, \hat{p}$

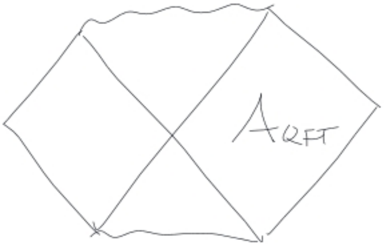
Constructed $\hat{A} \curvearrowright \mathcal{H} \otimes L^2(\mathbb{R})$

$$= \left\{ \hat{a} \in A \otimes B(L^2(\mathbb{R})) \,\middle|\, [\hat{a}, \overset{K\otimes 1}{-1\otimes\hat{x}}] = 0 \right\}$$

Thm: A type $\mathrm{III}_1 \longrightarrow \hat{A}$ type II

$$\tau(\hat{a}) = (\langle\Psi|\otimes\langle 0|_p) e^{\frac{\hat{x}}{2}} \hat{a} e^{\frac{\hat{x}}{2}} (|\Psi\rangle \otimes |0\rangle_p)$$

BHs



type $\mathrm{III}_{(1)}$

$G_N = 0$: $\mathcal{H}_{QFT}$, $\mathcal{A}_{QFT}$

$O(G_N)$: Couple $\mathcal{H}_{grav} \supseteq \mathcal{H}_{mass}$

$$\mathcal{H}_{mass} \cong L^2(\mathbb{R})$$

Focus on $\mathcal{H}_{QFT} \otimes \mathcal{H}_{mass}$ not all states phys.

$\mathcal{A}_{QFT} \otimes \mathcal{B}(\mathcal{H}_{mass})$ not all ops phys.

EE: $\Delta M = \int_{ext} T_{\mu\nu} \partial_t^\mu \, d\Sigma^\nu$

HH: 

$\langle \phi \phi \rangle \sim$ an. cont.
KMS

Physical alg.:

$$A_{phys} = \Big\{ \hat{a} \in A_{QFT} \otimes \mathcal{B}(\mathcal{H}_{mass}) \ \text{s.t.}$$

$$\Big[\hat{a}, \underbrace{\int T_{\mu\nu} \partial_t^\mu \, d\Sigma^\nu}_{\text{needed to be } K_\psi.} - \widehat{\Delta M}\Big] = 0 \Big\}$$

$$\frac{\kappa}{2\pi} K_{\psi_{HH}}$$

$\mathcal{A}_{phys.}$ is type II.

Takesaki's formula for τ.

Given $|\Phi_{QFT}\rangle \otimes |f_{mass}\rangle$

Define $\rho_{\Phi,f} \in (\mathcal{A}_{phys})_+$,

$$\tau(\rho_{\Phi,f}\,\hat{a}) = \langle \Phi \otimes f|\ \hat{a}\ |\Phi \otimes f\rangle$$

$$-\tau\left(\rho_{\Phi,f}\log\rho_{\Phi,f}\right)$$

$$= S_{A_{QFT}}\left(\Phi \,\|\, \Psi_{HH}\right) + \frac{\kappa}{2\pi}\langle \Delta M \rangle_f \quad \int dx\, x\, |f(x)|^2$$

$$- \int dx\, |f(x)|^2 \log|f(x)|^2$$

$$+ \text{const}$$

2206.10780

2306.01837

ΔS consistent $\quad O(G_N) \quad O(1)$

w/ $S = \frac{A}{4G_N} + \text{matt.}$

Interpretation

Akers & JS: For $\rho \in \mathcal{A}_{phys} \swarrow$ proj.

$$\mathbb{1}_{QFT} \otimes \Pi_{mass}$$

ΔS has a state-counting interpretation.

ρ, σ micro $\mathcal{H}_\rho\ \mathcal{H}_\sigma \leftarrow$ support

$$\underbrace{\mathcal{H}_\rho \hookrightarrow \mathcal{H}_\sigma \otimes \mathbb{C}^n}_{\text{unitary ops in } \mathcal{A}_{phys}} \quad \text{iff } n \geq e^{\Delta S}.$$

Lots of Q's:

→ Stat mech?

→ General BHs

→ type I?