ANOMALIES AND PERSISTENT ORDER IN THE χGN MODEL AT FINITE DENSITY

BASED ON 2203.07451 AND 2312.13756

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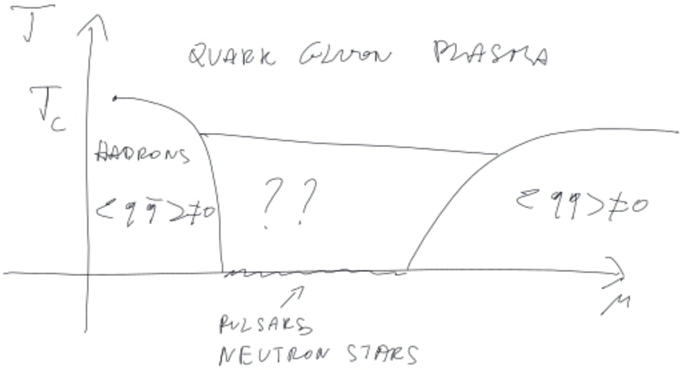
- MOTIVATION AND INTRODUCTION (LARGE N)
- FINITE N, $T=0$
- FINITE N, $T\neq 0$



COLOR SUPERCONDUCTIVITY

$$\langle qq\rangle \sim e^{-N}$$

$$\langle \bar{q}q\rangle \sim e^{-N^2}$$

$$\langle \bar{q}q\rangle = \Lambda(x)$$

$$\Lambda(x) = |\Lambda|\, e^{i\vec{q}\cdot\vec{x}}$$

CHIRAL SPIRAL

$$|\vec{q}| \sim \mu$$

DERYAGIN, GRIGORIEV, RUBAKOV, 1992

$$\mathcal{L}_{GN} = \bar{\psi}_k i\partial\!\!\!/ \psi_k + \frac{g^2}{2}(\bar{\psi}_k \psi_k)^2$$

$G = O(2N)$

$\langle \bar{\psi}\psi \rangle \neq 0$

GAPPED, UV-FREE, $\mathbb{Z}_2 \to \emptyset$

$$\mathcal{L}_{\chi GN} = \bar{\psi}_k i\partial\!\!\!/ \psi_k + \frac{g^2}{2}\left[(\bar{\psi}_k \psi_k)^2 (\bar{\psi}_k \gamma_3 \psi_k)^2\right]$$

$G = U(N)_V \times U(1)_A$

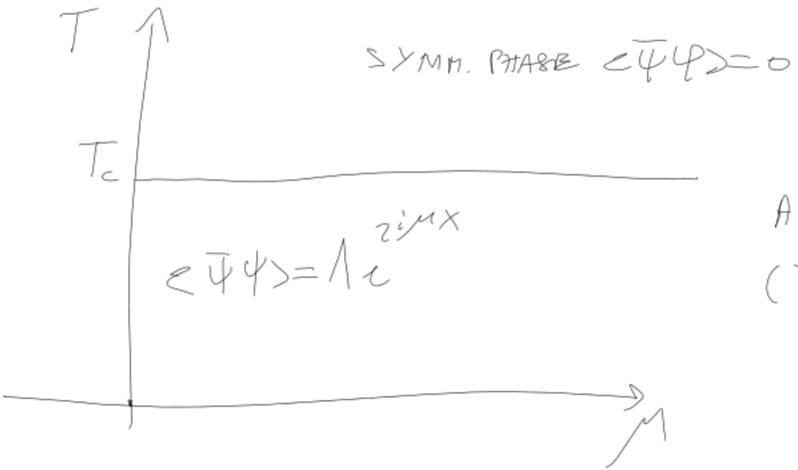
UV-FREE, GAPLESS

FOCUS

LARGE N

[2000–2009] THIES AND COLLABORATORS

χGN



AIM: EXTEND AT FINITE N

$(T=0,\ T\neq 0)$

LARGE N XGN $U(1)_A \to \mathbb{Z}_N^A$ (ARTIFACT OF LARGE N)

$\langle O \rangle \neq 0$ IMPOSSIBLE IN 2d

STRICT ORDER

$$\langle O(x)\, O(0) \rangle \xrightarrow[x\to\infty]{} \text{Const.} + x^{-2\Delta_0} + \dots \qquad d > 2$$

$$\langle O(x)\, O(0) \rangle \xrightarrow[x\to\infty]{} x^{-2\Delta_0(N)}\, h(u)$$

(QUASI LONG-RANGE ORDER)

$N \to \infty$ FIRST. $\Delta_0(N) \to 0$

(BKT)

$\langle O($

- FINITE N, $T=0$ $\quad \mathbb{R}^2$ $\quad U(N)_V \times U(1)_A$

EUCLIDEAN $\quad \psi_{\pm}$ WEYL

$$\mathcal{L}_{XGN} = i\psi^\dagger_{+a}\partial_-\psi_{+a} + i\psi^\dagger_{-a}\partial_+\psi_{-a} + \frac{\lambda_s}{N}\left|\psi^\dagger_{-a}\psi_{+a}\right|^2 - \frac{\lambda_v}{N^2}\left(\psi^\dagger_{+a}\psi^a_+\right)\left(\psi^\dagger_{-a}\psi_{-a}\right)$$

$a = 1, \dots, N$

$$\mathcal{L}_\mu = \mu\left(\psi^\dagger_{+a}\psi_{+a} + \psi^\dagger_{-a}\psi_{-a}\right)$$

$$\mathcal{L}_{XGN} \longrightarrow \mathcal{L}_{XGN} + \mathcal{L}_\mu$$

NON-ABELIAN BOSONIZATION

$U(N)_1$ WZW

$$\mathcal{L}_{XGN} \longrightarrow \mathcal{L}^{WZW}_{SU(N)_1}(V) + \frac{1}{2}(\partial\varphi)^2 + \frac{\lambda}{N} J_+^a J_-^a + \frac{\lambda'}{N} J_+ J_-$$

$V \in SU(N)$ $\qquad$ $SU(N)$ $\qquad$ $U(1)$ $\qquad$ $J_\pm \sim \partial_\pm \varphi$

λ MARGINALLY RELEVANT

λ' EXACTLY MARGINAL $\qquad$ $\lambda = 2\lambda_s, \; \lambda' = \lambda_v + \lambda_s$

$$\mathcal{L}_\mu \longrightarrow \mu\sqrt{N}\,(\partial_+ + \partial_-)\varphi \Rightarrow \langle \partial_x \varphi \rangle \neq 0$$

CHIRAL SPIRAL $\forall N$

$\mu' \sim \mu$ $\qquad$ $\psi^\dagger_{La}\psi_{Lb} = V_{ab}\, e^{i\varphi}$ $\qquad$ $\langle \psi^\dagger_a \psi^a(x)\, \psi^\dagger_a \psi^a(0) \rangle \xrightarrow[x\to\infty]{} |\mathrm{Tr}\,V|^2 e^{2i\mu' x} \cdot |x|^{-\frac{2}{N}\left(1+\frac{\lambda'}{2\pi N}\right)}$

$\mathrm{Tr}\, U \neq 0 \Rightarrow \mathbb{Z}_N^A$ $\qquad U \to g_L^\dagger U g_R \qquad g_{L,R}$

- $SU(N)_K$ HAVE A MIXED 't ~~HOOFT~~ ANOMALY

$PSU(N)_V \times \mathbb{Z}_N^A$ UNLESS $K = 0 \mod N$

VACUUM CANNOT BE TRIVIALLY GAPPED.

- SEMICLASSICALLY, ONE FINDS N VACUA

$\mathrm{Tr}\, U \neq 0 \Rightarrow$

N VACUA

- $N=2$ $\quad \mathrm{Tr}\, U$ CAN BE COMPUTED

$\Downarrow$

2 VACUA

FINITE N, $T \neq 0$ $\quad M = \mathbb{R} \times S^1_\beta$

TWO SPIN STRUCTURES

$B = \mathrm{SPIN}(2N)_1$ WZW

$F = N$ FREE DIRAC FERMIONS

$$F = \frac{B \times Arh}{\mathbb{Z}_2^L}$$

TQFT $Z_{ARF} = \begin{cases} 1 & \text{(EVEN S.S.} \\ -1 & \text{ODD S.S.} \end{cases}$

FINITE N, FINITE T

'T HOOFT ANOMALY $\quad U(1)_p \times U(1)_W$ $\quad \phi \to \phi + \alpha_p R \quad \alpha_p = \pi$

$\tilde{\phi} \to \tilde{\phi} + \frac{2}{R}\alpha_W \quad \alpha_W = \pi$

$\mathbb{C}$

C, P, W $\quad \mathbb{Z}_2$ 'T HOOFT ANOMALY

D_8 GROUP (FERMIONIC THEORY)

NS FERMION $\quad D_8$ IS LINEARLY REALIZED

R FERMION $\quad D_8$ IS PROJ. $\quad$ " $\quad$ ANOMALY

$\psi^\dagger\psi$ TOO HARD

$\det \psi_e^\dagger \psi_b = O_F \Rightarrow O_B = 1\, e^{i N \frac{\varphi}{R}}$

M GAP OF $SU(N)_1$ $\quad T \ll M$

$|x| M \gg 1$

$$\langle O_R(x)\, O_R(0)\rangle \underset{x\to\infty}{\sim} \begin{cases} e^{2i\mu x} & R \\ e^{-\frac{|x|}{NT}}\, e^{2i\mu x} & NS \end{cases}$$

· LARGE N

HS FIELD Δ

$$F = \frac{1}{\lambda_S} \int d^2x \, |\Delta(x)|^2 - \mathrm{tr} \log\left(i\partial\!\!\!/ + \Delta(x) P_+ + \Delta^*(x) P_-\right)$$

$N_c \gtrsim 1000 \, N_h$

ANSATZ : $\Delta(x) = M e^{2iqx}$

MINIMIZE F

$q = 0$ OR μ

$M = 0$, or Λ

$\langle \bar{\Psi}\Psi \rangle = \Lambda$

NS FERMIONS $T > T_c$

$M = 0$

R FERMIONS: $M \neq 0$ $\forall T$