Regge trajectories in N=4 SYM

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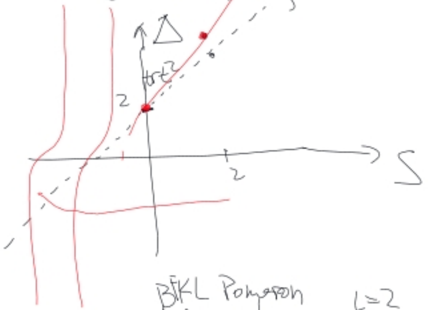
$m^2 \longrightarrow \Delta$

$J \longrightarrow S$



$\mathrm{tr}\, D_+^S Z^L \quad L=2$

$\Delta = S + L + \gamma(S) \quad \gamma(S) = g^2 H_1(S)$

pole at $S=-1$

$L=3$ 4 loops

$\gamma^{\text{1-loop}} = g^2 H_1(S/2)$

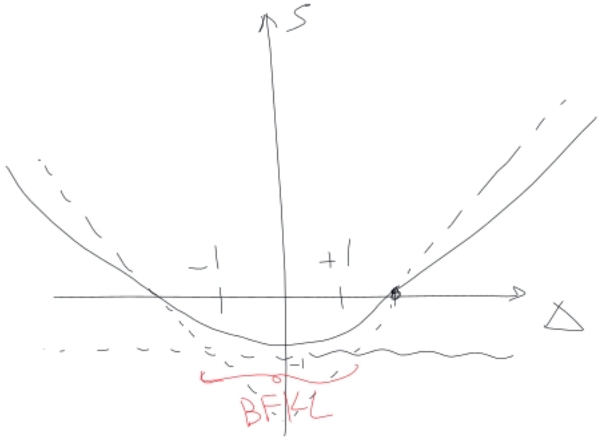
pole at $S=-2$

$\omega = S + L - 1 \quad \omega \to 0$

BFKL Pomeron eigenvalue

$$\Delta^{L=2} = \# \frac{g^2}{\omega} + \# \frac{g^4}{\omega^2} + \dots \left(\frac{g^2}{\omega}\right)^n = ?$$

$\frac{1}{\omega} \sim \ln S$



Pomeron

$$S(\Delta) = -1 + g^2 \underbrace{\left(\psi\left(\frac{1-\Delta}{2}\right) + \psi\left(\frac{\Delta+1}{2}\right) + 2\gamma\right)}_{\chi(\Delta)}$$

$$\omega = \chi(\Delta) + g^2 \chi^{(1)}(\Delta) + g^4 \ldots$$

LO

NLO (9 year to compute)

NNLO $\longrightarrow$ known in $N=4$

what is going on in general L?

$\underbrace{N \ldots N}_{L/2 \text{ times}} LO$ $\qquad N = nn$

$\mathcal{N}=4$ Integrability

$$\Psi(r,\theta,\varphi) = \Psi_\theta(\theta)\,\Psi_\varphi(\varphi)\,\Psi_r(r)$$

$$\Psi\left[\quad\right] = \prod_{i=0}^{\infty} Q_{A_i}(u_i) \qquad A_i \to \text{„polarizations"}\ \ psu(2,2|4)$$

- 2^8 different Q-functions $\quad$ 4+4 elementary

$Q_i(u)\quad i=1\ldots4 \quad \longleftrightarrow su(2,2) \quad AdS \quad$ com. charges

$P_a(u)\quad a=1\ldots4 \quad \longleftrightarrow su(4) \quad S \quad$ R-sym

$Q_1 \sim u^{\Delta+S+S_\perp} \qquad P_a \sim u^L$

$Q_2 \sim u^{\Delta-S-S_\perp}$

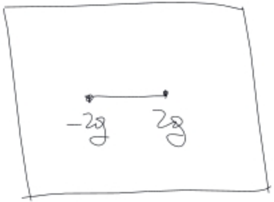
Schrodinger Ψ_1, Ψ_2 $\quad$ $\underbrace{\Psi_1\Psi_2' - \Psi_2\Psi_1'}_{Wr} = 1$

(A) Wr $\to$ SWr

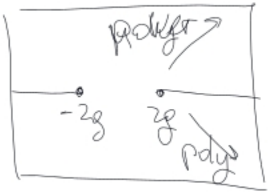
$$\sum_a P_a(u)\, P^a(u+i) = \sum_i Q_i(u)\, Q^i(u+i)$$

+ 3 more $\quad$ PSU(2,2|4)

(B) Analyticity $\quad g = \frac{\sqrt{\lambda}}{4\pi}$



P_a



Q_i

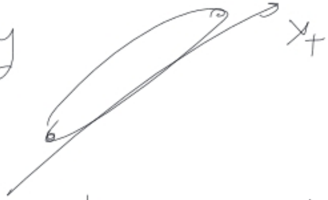
Conjecture:

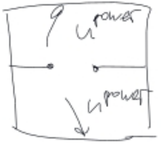
1-to-1

Local prim. op

ABJM, AdS_3, γ-β-def, 

$\Delta(S)$ $\quad \mathrm{tr}\, Z D_+^S Z \longrightarrow$ light-ray



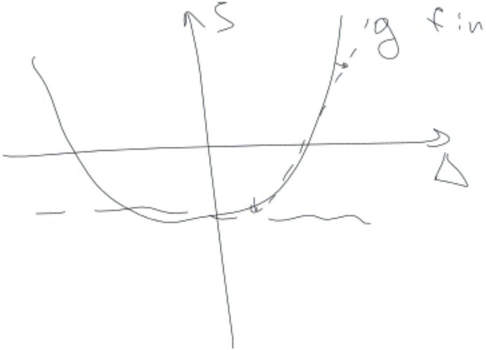


$Q_1 \; Q_3$

$\longrightarrow e^{2\pi i u}$ $\quad u^{\text{power}}$

$\Longrightarrow$ S is no longer integer
is allowed

numerically
70 digits



$L>2$: ∞ many trajectories

need analytic control

ABA

- BES eq. g^{2L+4} (ABA) DT
- Asym Baxter-Bethe Ansatz (ABBA) HT

simplification $\quad Q_i^{\circlearrowleft} = \overset{\text{monodromy}}{\omega_{ij}}\, Q^j \qquad P_a^{\circlearrowleft} = \mu_{ab} P^b$

DT : $\omega_{43} \sim g^{-2L-4}$ $\qquad x_k^{\pm} + \frac{1}{x_k^{\pm}} = \frac{u_k \pm i/2}{g} \qquad Q_4^{\circlearrowleft} = \omega_{43} Q_3$

Linear RH

$$p_k = \frac{1}{i} \ln \frac{x_k^+}{x_k^-} \qquad \varepsilon_k = 2ig\left(\frac{1}{x_k^+} - \frac{1}{x_k^-}\right) \qquad \mu_{12}(u_k + i/2) = 0$$

$$E_k = \sqrt{1 - 16 g^2 \sin^2 p_k/2}$$

HT : $\omega_{42} \sim g^{-L-2}$ $\qquad \omega_k = 2g \sin p_k/2 \qquad p_k = \frac{1}{i} \ln z_k^2 \qquad \omega_k = ig\left(z_k - \frac{1}{z_k}\right)$

once you know z_k $\Longrightarrow$ all P's Q's can be computed

$$P_1(u) = \sigma_0\, x^{-\frac{L}{2} - 1} \qquad x + \frac{1}{x} = \frac{u}{g}$$

$$\log \sigma_0 = \oint dx \oint dy\, \frac{1}{z - x}\, \frac{1}{y - z_i} \ln \frac{\Gamma(1 + iu(x) - iu(y))}{\Gamma(1 - iu(x) + iu(y))}$$

$z_k = ?$

$$\underbrace{(iz_k)^{2L+4}}_{e^{ipL}} = B_k \prod_{e=1}^{4} S_{ke} \qquad S_{ke} = \sigma_0(z_k, z_e)\, \frac{G_{ke}^{\frac{1+\Delta}{2}}\, G_{ke}^{\frac{1-\Delta}{2}}}{G_{ke}^{1}}$$

$$G_{ke}^{\delta} = \frac{\Gamma(\delta + i\theta_k - i\theta_e)}{\Gamma(\delta - i\theta_k - i\theta_e)} \qquad \frac{\theta}{g} = z + \frac{1}{z}$$

solve ABBA $\Rightarrow$ $w = \ldots\ldots + g^{L+1}$

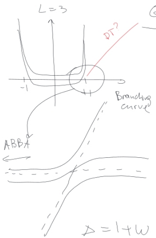
$$w = 4g\left(\cos\frac{\pi h_1}{L+2} + (h_1 \leftrightarrow h_2)\right) - 16 g^2 \chi(\Delta)\left(\frac{\sin^2\frac{\pi h_1}{L+2}}{L+2} + (h_1 \leftrightarrow h_2)\right) + \ldots g^{L+1}$$

$$1 \le h_1 < h_2 \le L+1$$
$$h_1 + h_2 \in 2\mathbb{Z}$$

$\underline{L=2}$ $\quad h_1 = 1 \quad h_2 = 3 \qquad W = 0\,g + g^2 \chi(\Delta)$ ← QCD Pomeron!

$\underline{L=3}$ $\quad h_{\cancel{a}} = 1,3 \quad 2,4 \qquad w = \pm 2g - 4g^2\chi \mp \frac{2\pi^3}{3} g^3 + 24\left(\chi'' + \frac{7\zeta_3}{6} + \frac{\pi^2}{18}\chi\right) g^4$

$\underline{L=30}$ $\to$ 225 states, 31 terms in g



Gluing HT to DT

$$w = \Omega g$$
$$\Delta = 1 + g D$$

$$\Omega^3 + A\Omega^2 + B\Omega + C = 0$$

poly in D

$$\Omega_{1,2} = \pm 2 - \frac{8}{D} + \frac{96}{D^3} + O\left(\frac{1}{D^4}\right)$$

$$\Omega^3 - D\Omega^2 - 20\Omega + 4D = 0$$

$$\Delta = 1 + w - \frac{16g^2}{w^2} - \frac{64g^4}{w^4} + \quad g^6 \qquad g^8$$

$L=4 \implies \Omega^5 + \dots\dots = 0$

$$\Delta - 1 = 4g^2 \frac{3 \pm \sqrt{5}}{\omega}$$

$\mathrm{tr}\, Z^3 D_+^2$

$L=11$ $\qquad P_n = 32 - 2\gamma_n$

$\mathrm{tr}\, Z^L D_+^2$

Conclusions