Bootstrapping the Maldacena-Wilson line defect CFT

Based on:
- 2103.10440
- 2312.12550, 2312.12551 w/ C. Meneghelli

INTEGRABILITY

LOCALIZATION

SCATTERING AMPLITUDES

Gromado '18,
Canom-Huot et al '22
Chester, Dempsey, Pufu '23

$\mathcal{N}=4$ SYM

Binder, Chester, Pufu, Wang '19
...

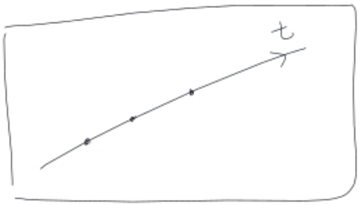
BOOTSTRAP

HOLOGRAPHY

Rastelli, Zhou '16
...

Simpler model: MW line defect

$\mathbb{R}^{1,3}$



$$\langle \mathrm{tr}_R \, P \; O_1(t_1) \dots O_n(t_n) \, e^{i\int A_t + \Phi^6} \rangle$$

BOOTSTRAP + INTEG.
+ INT. CONSTRAINTS

Cavaglià, Gromov, Julius, Preti '22
. . .
(+ Machine learning)

LOCALIZATION + INTEG.
+ LARGE CHARGE

Giombi, Komatsu '18,
. . .

ANALYTIC BOOTSTRAP

Giombi, Roiban, Tseytlin '17
Meneghelli, Liendo '18

↓

THIS TALK
(Holographic correlators at strong coupling)

1d HOLO CORRELATORS

$\to$ Superconformal kinematics (long mult., higher-point)

$\to$ higher loops (3 loops)

$\to$ operator degeneracy, dilatation op. @ $\lambda \to \infty$

D) SPECTRUM

$SU(N)$ $\quad N \to \infty$ $\quad \lambda = g_{YM}^2 N$, $\boxed{\lambda \to \infty}$

$\mathrm{Rep} = F$

$$OSP(4|4) \supset \underset{\substack{\downarrow \\ \text{4d conf.} \\ \text{group}}}{Sl(2)} \oplus \underset{\substack{\downarrow \\ \perp \text{ rot.}}}{su(2)} \oplus \underset{\substack{\downarrow \\ \text{R-symm}}}{SO(6)}$$

SUPERSTRING IN $AdS_5 \times S^5$

$$X^\mu = \Big(\underbrace{t, S}_{AdS_2} , \underset{\substack{\downarrow \\ AdS_5}}{x^{i=1,2,3}} , \underset{\substack{\downarrow \\ S^5}}{y^{a=1,\dots,5}} \Big)$$

$$\mathcal{L}_{AdS}(y, x, \psi) = (\partial y)^2 + \frac{1}{\sqrt{\lambda}}\left[(\partial y)^4 + \dots\right] + \frac{1}{\lambda}\left[(\partial y)^6 + \dots\right]$$

$$+ x, \psi$$

$(x, \psi, y) \longrightarrow$ 1/2-BPS MULT. D_1

$$m^2 = \Delta(\Delta - 1)$$

	Δ	SU(2)	SO(5)
y	$\Delta = 1$	1	5
ψ	3/2	2	4
x	2	3	1

$$\langle D_1 D_1 D_1 D_1 \rangle$$

- NO KK MODES
- $O\left(\frac{1}{\lambda^{\ell/2}}\right) \longleftrightarrow (\partial y)^{2\ell+2}$

ALL FIELDS $= \Pi\ (\partial, y, x, \varphi)$

1) ½-BPS

- $D_k \sim (D_1)^k \qquad [0,k]$ of $so(5)$
- 1 $\forall k$

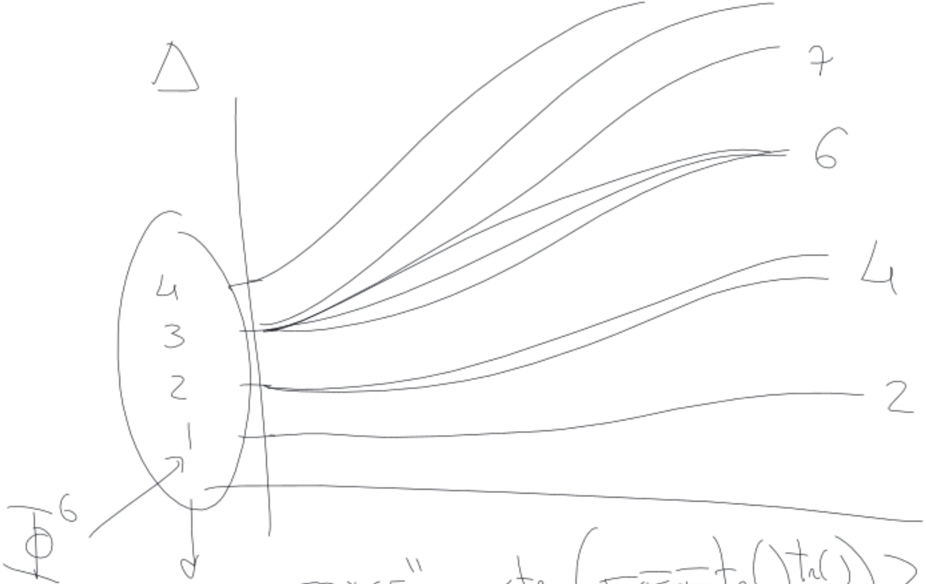
2) $\mathcal{L}^{\Delta}_{S,[a,b]}$

- $\Delta > \Delta_{U.B.}$
- Degeneracy

e.g. $\Delta = 4$: $\quad y^4, \quad \partial^2 y^2 + \cdots$

Δ	2	3	4	5	6	7	8	9	10
#	1	0	2	0	4	1	9	3	21

$\#(\Delta) \sim \Delta^2$



"OPEN-TRACE" $\langle \mathrm{tr}\left(FFF\, \mathrm{tr}_2(\,)\, \mathrm{tr}(\,) \right) \rangle$

$$OT \times OT \sim OT + \left(\frac{HT}{N^{\#}} \right)$$

$\langle D_1 D_1 D_1 D_1 \rangle$ at $O\left(\frac{1}{n^2}\right) \rightsquigarrow 3$ loops

position space ansatz

$$D_1 \times D_1 = D_2 + \sum_{\Delta} \mathcal{L}^{\Delta}_{0,(0,0)}$$

- "y^2" $\to \Delta_{y^2}$, $\to C_{D,D,y^2}$ at $O\left(\frac{1}{\lambda^2}\right)$

- Dilatation op.

2) BOOTSTRAP

$$G^{(\ell)}(x) = \sum_i R_i(x)\, T_i(x)$$

poles @ $x = 0, 1$

harmonic polylogs

$$H_{(a_1 \dots a_m)}(x) = \int_0^x dt\, \frac{1}{a_1 - t}\, H_{(a_2 \dots a_m)}(t) \qquad a_i \in (0,1)$$

m = TRANSC.

ORDER $\frac{1}{\lambda^{\ell/2}}$: MAX TRANSC. $= \ell$

- AdS unitarity

$$g_\Delta^{(1d)}(x) = x^\Delta \, {}_2F_1(\ldots)$$

$$\Delta = \underset{\substack{\cap \\ 2\mathbb{N}}}{\Delta^{(0)}} + \frac{1}{\sqrt{\lambda}} \gamma_\Delta^{(1)} + \ldots \qquad x^\Delta \sim x^{\Delta^{(0)}} \left(1 + \frac{1}{\sqrt{\lambda}} \gamma_\Delta^{(1)} \log x \right)$$

$$G^{(\ell)}(x) = \sum_{k=0}^{\ell} \underbrace{G^{(\ell)}_{\log^k}(x)}_{\text{s.v. @ } x=0} \log^k x$$

$G^{(\ell)}_{\log^k}$ for $k \geq 2$ depend on prev. orders CFT data

- CROSSING

$\checkmark \longrightarrow$ HPL

$x \longleftrightarrow 1-x$

- OPE

- REGGE LIMIT

$$\gamma_{\Delta}^{(p)} \sim \Delta^{p+1}$$

3) MIXING

$$G^{(1)}_{\log^1}(x) = \sum_{\Delta} \langle a^{(0)}_{\Delta} \gamma^{(1)}_{\Delta} \rangle g^{(\mathrm{id})}_{\Delta}(x)$$

For $\Delta = \overline{\Delta}$ $\exists$ 2 operator

$$\langle a^{(0)} \gamma^{(1)} \rangle_{\overline{\Delta}} = \left(C^{(0)}_{11O_1}\right)^2 \gamma^{(1)}_{O_1} + \left(C^{(0)}_{11O_2}\right)^2 \gamma^{(1)}_{O_2}$$

$$G^{(2)}_{\log^2}(x) = \sum_{\Delta} \langle a^{(0)}_{\Delta} \left(\gamma^{(1)}_{\Delta}\right)^2 \rangle g_{\Delta}(x)$$

$$\gamma^{(1)}_{\Delta}(s,[a,b]) = C_{osp(4|4)}(s,a,b,\Delta)$$

$$\langle \mathcal{D}_p \mathcal{D}_p \mathcal{D}_q \mathcal{D}_q \rangle$$

1 loop ✓

2 loops ✓

3 loops ?

$$\langle a^{(0)}_{\Delta} \left(\gamma^{(2)}_{\Delta}\right)^2 \rangle$$

$\Delta = 6$: $\partial^4 y^2,\ \partial^2 y^4,\ \partial^2 y^4,\ y^6$

$$O_\alpha = \sum_\beta M_{\alpha\beta}\, \tilde{O}_\beta$$

EIGENSTATES

$$\gamma^{(2)}_\Delta \longrightarrow \Gamma^{(2)}_\Delta = \begin{pmatrix} \Gamma^{(2)}_{2\to2} & 2\to4 & 2\to6 & \cdots \\ \vdots & 4\to4 & \cdots & \\ \vdots & & \ddots & \end{pmatrix}$$

$$\frac{1}{\lambda^{\ell/2}} \rightsquigarrow y^{2\ell+2} \qquad \delta L = 2\ell - 2$$

$$\langle D, X_{L=3}\rangle = 0$$

$$\Gamma^{(1)}_{\triangleright} = \begin{pmatrix} \Gamma^{(1)}_{2\to 2} & & \\ & \Gamma^{(1)}_{4\to 4} & \\ & & \ddots \end{pmatrix}$$

$$\#_{L=2}(\triangleright) = 1$$

$$\#_{L=4}(\triangleright) = \Delta^2$$

$$\Gamma^{(2)}_{\triangleright} \quad \begin{pmatrix} \Gamma_{2\to 2} & \Gamma_{2\to 4} \\ \Gamma_{4\to 2} & \Gamma_{4\to 4} \\ \vdots & \end{pmatrix} \quad L=2$$

$$L=2$$

$$\langle a^{(0)} \left(\gamma^{(2)}\right)^2 \rangle = \sum_{\alpha,\beta,\gamma} C^{(0)}_{11 O_\alpha} \Gamma^{(2)}_{\alpha\beta} \Gamma^{(2)}_{\beta\gamma} C^{(0)}_{11 O_\gamma}$$

O_α $L=2$

$$\langle D, D, D_2\, \mathcal{Z}^{\Delta_{ext}} \rangle$$

$$\langle D, D, \mathcal{Z}^{\Delta}\, \mathcal{Z}^{\Delta'} \rangle$$

$\Delta_{ext} \leq 16$

$\Delta_{exch} \leq 18$

1 - loop

$$\gamma^{(2)}_{\Delta} \sim H^{(2)}_{\Delta}$$

OUTLOOK

- HPL ansatz: "SV" HPLs $(z \to \bar{z} \to x)$

  Veneziano (Alday et al)

- Mixing problem