New Approaches to Adjoint QCD$_2$

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Based on arXiv: 2311.09334, 2406.17079

4D Yang-Mills, Real world QCD

Adjoint QCD$_2$

2D Yang-Mills, (Fundamental) QCD$_2$

4D Yang-Mills

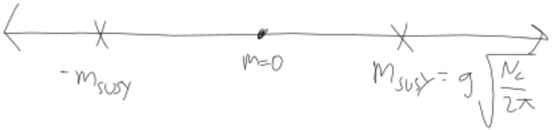
$$S=\int d^4x \, \mathrm{tr}\left[-\frac{1}{2g^2}F_{\mu\nu}F^{\mu\nu}\right]$$

Interesting features:

- mass gap
- confinement of test quarks

Adjoint QCD_2

$$S=\int d^2x \, \mathrm{tr}\left[-\frac{1}{2g^2}F_{\mu\nu}F^{\mu\nu}+i\bar{\psi}\not{D}\psi-m\bar{\psi}\psi\right]$$



Surprises:

1. Mass gap $(m=0)$
2. Deconfinement $(m=0)$
3. SUSY at $m\geq g\sqrt{\frac{N_c}{2\pi}}$

Surprise 1: Mass gap @ $m=0$

IR: 2D gauge theory $\longrightarrow$ WZW model

$$(G, \lambda) \longrightarrow \frac{so(\dim \lambda)_1}{g_{I(\lambda)}}$$

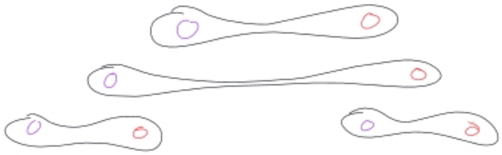
Dynkin index

$$c_{IR} = \underbrace{\frac{\dim \lambda}{2}}_{c_{UV}} - \frac{I(\lambda) \cdot \dim g}{h^\vee + I(\lambda)}$$

$$\lambda = adj \Rightarrow I(\lambda) = h^\vee, \ \dim \lambda = \dim g \Rightarrow c_{IR} = 0$$

Surprise 2: Deconfinement @ $m=0$

Fundamental matter $\longrightarrow$ QCD string can break



$\Rightarrow$ Wilson loop has perimeter law

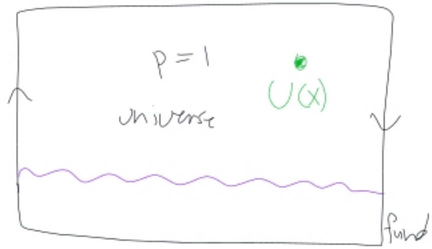
Adjoint matter $\longrightarrow$ String cannot break $\rightarrow$ Area law for Wilson loop indicates confinement

$m=0$: Perimeter law, deconfinement

Universes and String Tension

Adjoint has N_c-ality 0 in $SU(N_c)$, so N_c-ality of bg. flux cannot change!

$p=0$
universe



Deconfinement $\longrightarrow$ Vacua in each universe are degenerate

$m=0$: Known from non-invertible symmetries (2^{N_c-1} deg. vacua)

Confinement $\longrightarrow$ $\sigma_p = \frac{1}{L}\left(E_p - E_0\right)$

Surprise 3: (1,1) SUSY @ $m = g\sqrt{\frac{N_c}{2\pi}}$

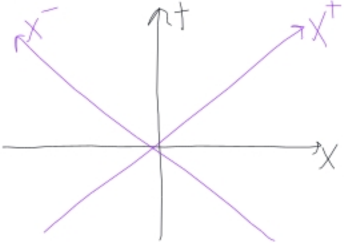
From

$$S = \int d^3x \, \mathrm{tr}\left[-\frac{1}{2g^2} F_{\mu\nu} F^{\mu\nu} + i\bar{\Psi} \not{D} \Psi - g\sqrt{\frac{N_c}{2\pi}} \bar{\Psi}\Psi\right]$$

SUSY is non-obvious

Expectation:
- $\rightarrow$ unbroken in $p=0$ universe
- $\rightarrow$ broken in $p \neq 0$

"Old" Approach: Light cone Quantization



Integrate out gauge field, one of the fermion components

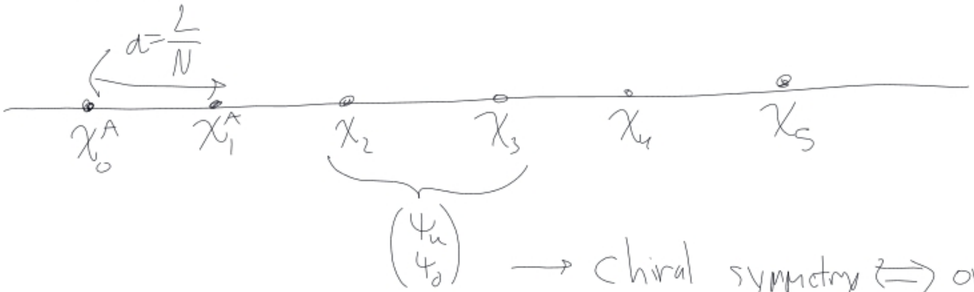
$$P^+ = \int dx^- \, \mathrm{tr}\left(\frac{i}{2}\psi \partial_- \psi\right), \quad P^- = \int dx^- \left(-\frac{g^2}{4} J_+ \frac{1}{\partial_-^2} J_+ + \frac{i m^2}{4} \psi \frac{1}{\partial_-} \psi\right)$$

From this:

1. Mass gap @ $m=0$ $\quad 0.95\, g\sqrt{N_c}$
2. Deconfinement @ $m=0$
3. SUSY @ m_{susy}

BUT: only $p \geq 0$ universe

New Approach #1: Lattice Hamiltonian



$\rightarrow$ chiral symmetry $\Longleftrightarrow$ one-site translation

$\rightarrow$ no doublers

Questions:

1) Hilbert space on links?

2) On-site representation G

Gauge-Invariant Hilbert Space

1) Links $\longrightarrow$ gauge connection $\longrightarrow$ particle on G

Peter-Weyl thm: $L^2(G)$ = matrix elements of irreps

$|r_n; m_{n,L}\, m_{n,R}\rangle$

2) $\{\chi^A_m, \chi^B_n\} = \delta_{mn}\delta^{AB}$ $\hookrightarrow$ spinor of $SO(N\cdot\dim G)$

$G^{\otimes N} \hookrightarrow SO(N\cdot\dim G)$

$\downarrow$

$2^{\frac{N}{2}\mathrm{rk}\,G}(R, R, \ldots, R)$ $R =$ 

$SU(2)$: $R = 2$

$SU(3)$: $R = 8$

$SU(4)$: $R = 64$

Lattice Hamiltonian

$$H = \sum_{n=0}^{N-1} \Big[\underbrace{\frac{g^2 a}{2} L_n^A L_n^A}_{C_2(r_n)} - \frac{i}{2}\left(a^{-1} + (-1)^n m\right) \chi_n^A U_n^{AB} \chi_{n+1}^B \Big]$$

link connection

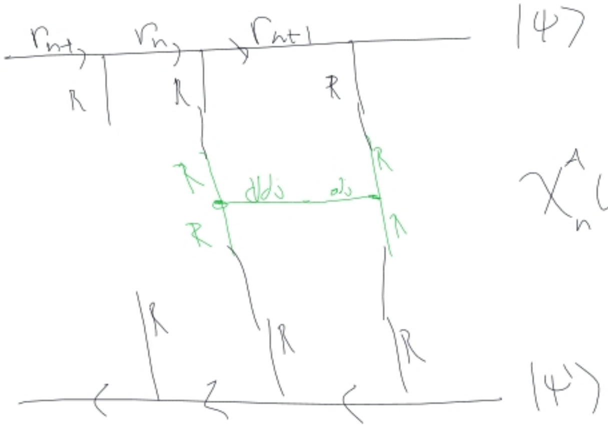
- Continuum limit ✓
- Symmetries
  - Fermion parity $\chi \to -\chi$
  - Charge conj. $\chi \to -\chi^T$ $U \to -U^*$
  - One-form center: N-ality on links
  - Chiral symmetry $(m=0)$: one-site translation

N-ality of R:

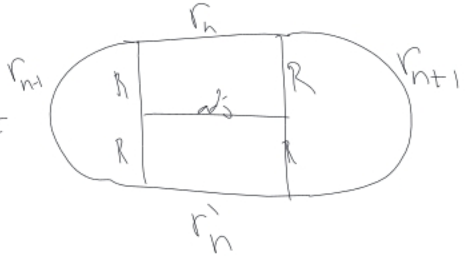
N_c even $\to \frac{N_c}{2}$

N_c odd $\to 0$

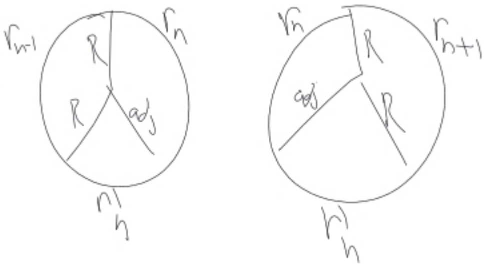
Matrix element



$$\chi_n^A U_n^{AB} \chi_{n+1}^B =$$



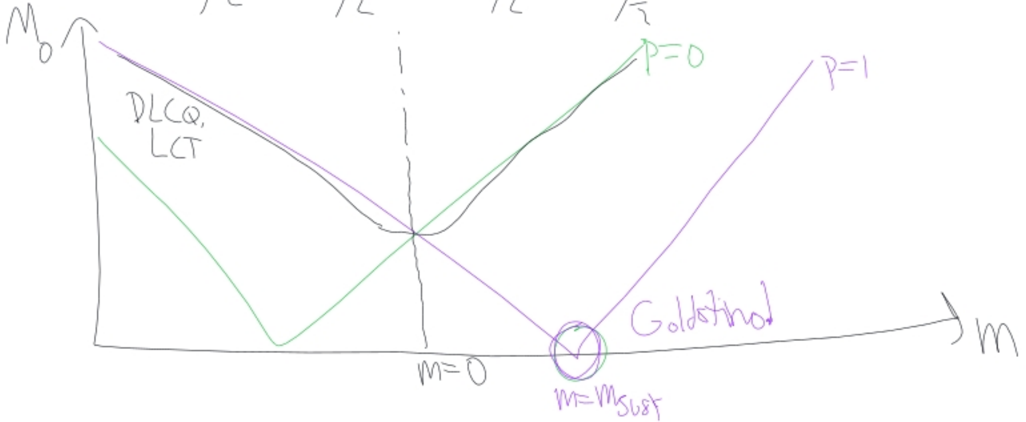
$\cong$



SU(2) results

For SU(2), $p=0$ @ $m \Leftrightarrow p=1$ @ $-m$

max spin < cutoff



"New" Approach #2: Small Circle EFT

Let $gL \ll 1$. With PBC for fermions.

circle length

What remains:

- Fluctuations of gauge field holonomy (rk G bosons q_a)
- Zero modes of fermion in Cartan (rk G fermions χ_a)

What is the effective QM?

$$\mathcal{L} = \frac{1}{2}\dot{q}_a\dot{q}_a + \underbrace{\frac{g^2 N_c}{4\pi}}_{\frac{1}{2}\left(g\sqrt{\frac{N_c}{2\pi}}\right)^2} q_a q_a + \frac{1}{2}\chi_a^T \dot{\chi}_a + \frac{m}{2}\chi_a^T \gamma_0 \chi_a$$

$$+ (gL)^2 B_{ab}\left[\left(\frac{3g^2 N_c}{4\pi} - \frac{m^2}{2}\right) q_a q_b + \frac{m}{2}\chi_a^T \gamma_0 \chi_b\right]$$

$$+ (gL)^{5/2} \cdots\cdots (\propto m)\, q\chi\chi$$

$$B_{ab} = \sum_{\vec{\alpha}>0} \alpha_a \alpha_b \sum_n \frac{1}{|2\pi n - \vec{a}_* \cdot \vec{\alpha}|^3}$$

$$+ (gL)^3 \cdots\cdots (\propto m)\, qq\chi\chi$$

Feature: 1) SUSY QM @ $m = m_{susy} = g\sqrt{\frac{N_c}{2\pi}}$

2) $m = 0 \Rightarrow$ decouple fermions $\longrightarrow$ $\deg = 2^{N_c - 1}$

Summary

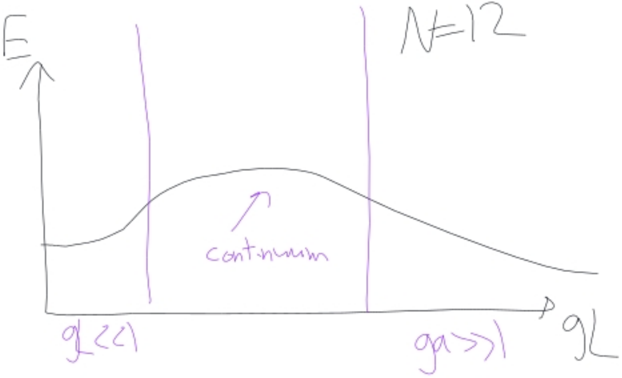
1) Adjoint QCD_2 is uniquely interesting — mass gap, deconfinement, SUSY

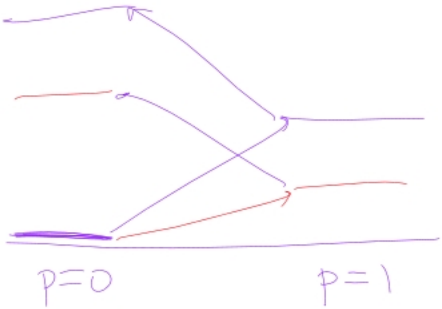
2) Lattice Hamiltonian → numerics $\forall$ universes

↳ $SU(3)$, ...

3) Small circle → deg., SUSY near $gL \ll 1$; all gL?

↳ more/all-orders


E
N=12
continuum
gL<<1
ga>>1
gL


p=0
p=1