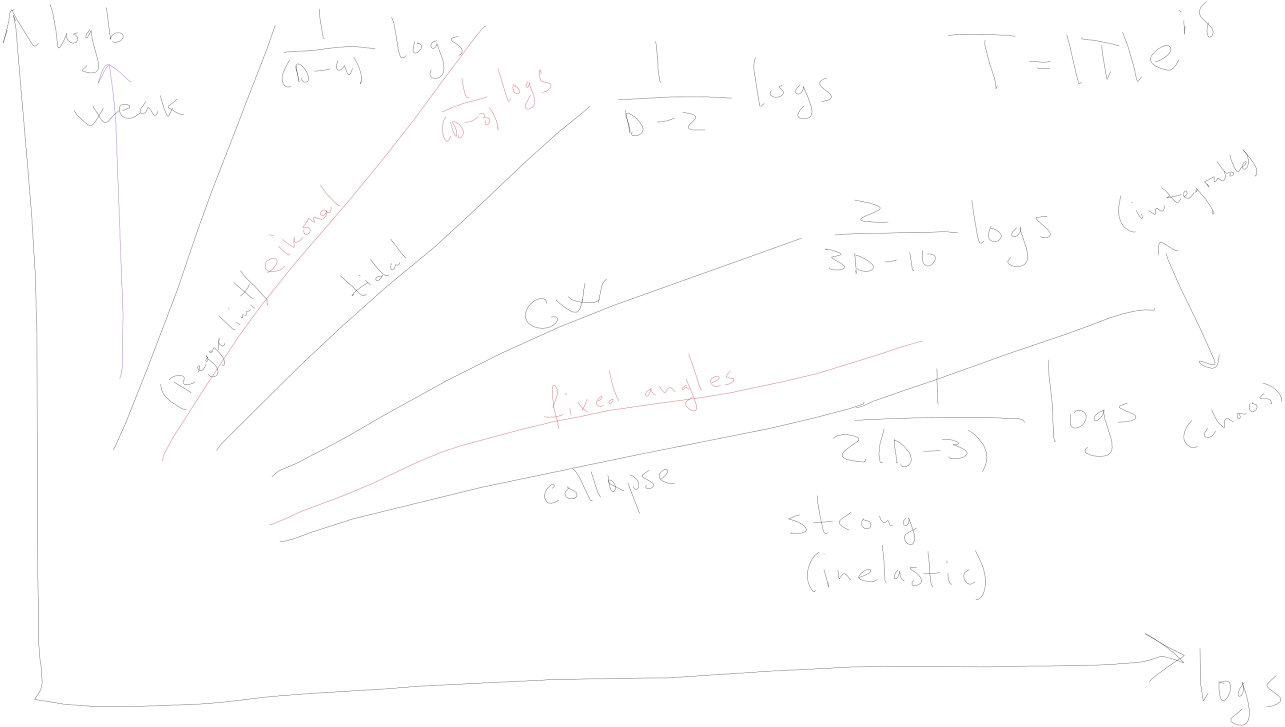

log b
weak
$\frac{1}{(D-4)} \log s$
$\frac{1}{(D-3)} \log s$
$\frac{1}{D-2} \log s$
$T = |T| e^{i\delta}$
$\frac{2}{3D-10} \log s$
(integrable)
(chaos)
$\frac{1}{2(D-3)} \log s$
(Regge limit)
eikonal
tidal
GW
fixed angles
collapse
strong
(inelastic)
log s

$$T(s,b) = e^{2i\delta(s,b)} - 1$$

$$\left| T_{eik}(s,t) \right| \sim s^{2 - \frac{(D-4)}{2(D-3)}}$$

$$t = -q^2$$

<u>Fixed angle:</u> $\Theta \sim \frac{2q}{\sqrt{s}} \ll 1$

<u>ACV saddle:</u>

$$q = \frac{G_N s}{b_*^{D-3}}$$

$$\Theta \approx \left(\frac{R_s}{b_*} \right)^{D-3} \ll 1$$

$$\delta_{GW} = i \left.\frac{G_N^3 s^2}{b^{3D-10}}\right|_{b=b_{ACV}} \simeq S_{BH}(\sqrt{s})\,\theta^{3D-10}$$

<u>HE FA</u>:

$$|T(s,\theta)| \simeq e^{-S_{BH}(\sqrt{s})\,f(\theta)}$$

$$f(\theta) \simeq \theta^{3D-10} + \dots \qquad \theta \ll 1$$

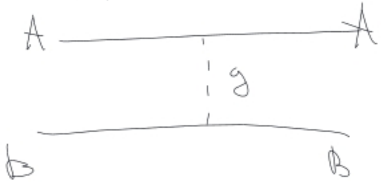
$$T(s,t) = \sum_{J=0}^{\infty} \overset{J^{d-3}}{\overset{\sim}{n_J}}\, \overset{\lesssim\, s^{2-d/2}}{f_J(s)}\, \left| P_J\left(1+\frac{2t}{s-4m^2}\right) \right|$$

$$S_J(s) = 1 + i\,\frac{(s-4m^2)^{\frac{D-3}{2}}}{\sqrt{s}}\, f_J(s)$$

$$|S_J(s)| \leq 1 \qquad s \geq 4m^2$$

- $D > 7$ PW absol.
- $D = 6, 7$ $\lim_{J_{max} \to \infty} \sum_{J=0}^{J_{max}}$
- $D = 5, 4$ (distributions)

(gravity)



$$T_\Psi(s) = \int_0^{q_0} dq\, q\, \Psi(q)\, T(s, -q^2)$$

suppress large J/b

$$\begin{cases} \Psi_{a,b}(q) \sim q^a \\ \Psi_{a,b}(q) \sim (q_0 - q)^b \end{cases}$$

$$T(s,t) = \underbrace{\sum_{J=0}^{J_*}}_{\text{nonpert. unitarity}} + \underbrace{\sum_{J_*}^{\infty}}_{\text{known/eikonal}}$$

$$s^{2-\frac{(D-3)}{2(D-2)}} \qquad \underbrace{s^{2-\frac{(D-4)}{2(D-3)}}}_{\text{Dominate}}$$


s
sub exp
crossing
unitarity
0
s_0

Pointwise

Smeared

Regge bound

$$|s|^{2-\frac{D-4}{2(D-3)}}$$

$$|s|^{2-\mathrm{Min}\left(1,\frac{a}{D-4},\frac{b+\frac{d-1}{2}}{D-2}\right)}$$

$$\lim_{|s|\to\infty}\frac{T}{|s|^2}=0$$

$$|T_\Psi| \lesssim |s|$$

crossing

$$\frac{T(s,t)}{s^2} = \frac{2}{\pi} \int \frac{ds'}{(s')^3}\, T_s(s',t)$$

crossing

$$\underbrace{-\frac{8\pi G_N}{t}}_{\text{Newton's pot.}} + \ldots = \frac{2}{\pi} \int_{s_0}^{\infty} \frac{ds'}{(s')^3}\, \underbrace{T_s(s',t)}_{\text{propagation through the shock}} \qquad \boxed{t \to 0}$$

eikonal saddle

Problem: crossing eikonal

$$T_S(s,t) = 2s\,(2\pi)^{\frac{D-2}{2}} \int_0^\infty db\, b^{D-3} \,(q/b)^{\frac{4-D}{2}}$$
$$J_{\frac{D-4}{2}}(bq)\, 2 \sin^2 \delta_{\text{tree}}(s,b)$$

$$\int_0^\infty \frac{dx}{x^2} \sin^2(x) = \frac{\pi}{2}$$

$$x = \frac{G_N s}{b^{D-4}}$$

The Cerulus-Martin bound

(lower bound on scatt at fixed angles)

$$|T(s,\theta)| \gtrsim e^{-\sqrt{s}\log s\, f(\theta)} \quad (\text{QFT})$$

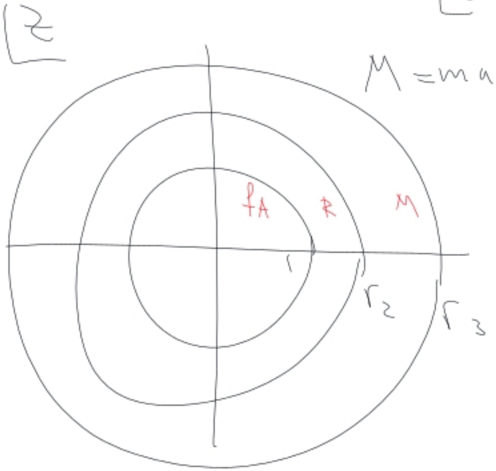
$$|T(s,\theta)| \sim e^{-S_{BH} f(\theta)} \sim e^{-(\sqrt{s})^{1+\frac{1}{D-3}} f(\theta)}$$

$\forall t(\sim s)$

- maximal analyticity
- Mandelstam repr. $|T(s,\theta)| \leq S^N$

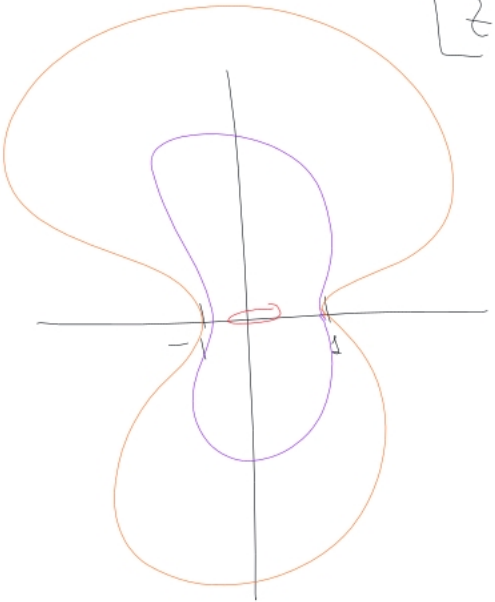
Regge $\leq$ (Fixed angle) $\times$ (Mandelstam)

z



$$M = \max_{|z|=r} f$$

$$M_{r_2} \leq M_1^{1-\frac{\log r_2}{\log r_3}} \, M_{r_3}^{\frac{\log r_2}{\log r_3}}$$



$z = \cos\theta$

~~Mandelstam rep~~

in gravity

Stringy features

$$g_s \ll 1 \Leftarrow M_s \ll M_{pl}$$

$$G_N = \# \frac{g_s^2}{M_s^{D-2}} = \frac{1}{M_p^{D-2}} \qquad \ell_s = \frac{1}{M_s}$$

The correspondence point: $R_s(S_*) = \ell_s$

$$M_{pl} = \frac{M_s}{g_s^{\frac{2}{D-2}}} \ll \sqrt{S_*} = \frac{M_s}{g_s^2} \qquad \begin{cases} S < S_* \\ b < \ell_s \log s\alpha' \end{cases}$$

"stringy gravity"

log b
log ℓ_s
stringy regime
GR
collapse
HP
log M_s
log M_s/g_s
log M_s/g_s^2
log s