Topics on QFT in AdS

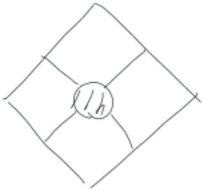
I. Flatspace limit of AdS (SK, Paulos, van Rees, Zhao, 2007.13745)

II. Mass gap & confinement in AdS (Copetti, Di Pietro, Ji, SK, 2312.09277)

III. Non-invertible symmetries & flat-space limit (Copetti, Córdova, SK, 2403.04835)

0. Intro

$$\xrightarrow[m,\ g\ \text{fixed}]{R_{AdS} \gg 1}$$



$$\langle O_1 O_2 O_3 O_4 \rangle \xrightarrow{R_{AdS} \gg 1} S(s,t)$$

Why?

• QFT in AdS : $\underbrace{\text{IR finite, symmetric}}_{S^d}$, $\exists$ asymptotic obs

$\mathbb{R}^{d,1}$

(1b) Conformal correlator : Analyticity is better understood

— Convergence OPE

⇝ Insight into non-perturbative analyticity of S-matrix? (in particular higher-pnt amplitudes)

Examples

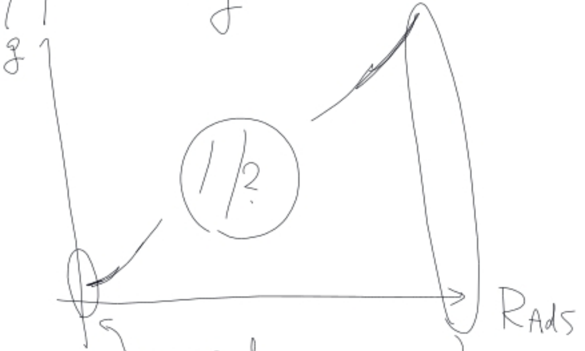
- Conformal Regge theory ⟷ Regge theory

Lightray operators ⟷ ?

- Higher pnt amplitudes / correlators
  and Regge theory

$\rightsquigarrow$
- Analytically continue quantum numbers
  which are not spin
- there's no dynamical pole in
  that quantum number

Asymptotically free theories in AdS



1. No phase transition
   (Neumann b.c. YM in AdS_4)
2. $\exists$ phase transition
   (Dirichlet b.c. YM)

1 $\rightsquigarrow$ Semi-classical Mass gap / confinement

– "Adiabatic continuity" [Argyres Unsal, Poppitz, —]

(Potential interplay with semiclassics renormalons, resurgence etc)

$\rightsquigarrow$ not much explored yet.

2 $\rightsquigarrow$ Confinement / Deconfinement transition

from bootstrap (existence & properties)

(The paper today by Guerrieri, De Cesare, Di Pietro, Serone)

I. Flatspace limit of AdS

<u>Goal</u>

- $\langle O_1 O_2 O_3 O_4 \rangle \xrightarrow{R_{AdS} \gg 1} S(s,t)$

  when & how this works

- $\sum_{\Delta, J} C_{\Delta, J} F_{\Delta, J}$ | $|S|^2 \leq 1$

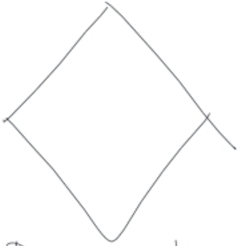
  Unitarity $C_{\Delta, J} \geq 0$

  Linear $\longrightarrow$ Non linear

- Map, $z, \bar{z} \longrightarrow s, t$

1. Kinematics

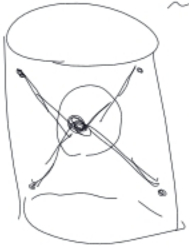
QFT in AdS $\xrightarrow[\text{keeping } m,\, g]{R_{AdS} \gg 1}$



- $m^2 R_{AdS} = \Delta(\Delta - d) \rightsquigarrow \Delta \sim m R_{AdS} \gg 1$

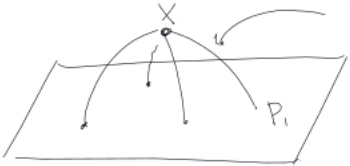
$\rightsquigarrow$ Compton wavelength $\ll$ AdS radius

$\rightsquigarrow$ Classical particle moving along geodesics



$\rightsquigarrow \langle O_1 O_2 O_3 O_4 \rangle \sim \hat{S} \times (\text{leg factors})$

• Assume this is correct & Derive $(z,\bar{z}) \to (s,t)$



$$\frac{1}{(P_1 \cdot X)^{\Delta}}$$

$$\int dX \frac{1}{(P_1 \cdot X)^{\Delta}} \frac{1}{(P_2 \cdot X)^{\Delta}} \frac{1}{(P_3 \cdot X)^{\Delta}} \frac{1}{(P_4 \cdot X)^{\Delta}}$$

$$X \cdot X = -1$$

$$\downarrow$$

$$S[X] = -\sum_i \Delta_i \log(-P_i \cdot X) + \lambda (X^2 + 1)$$

Saddle pnt : $\displaystyle \sum_i \Delta_i \frac{P_i}{(P_i \cdot X)} - 2\lambda X = 0$

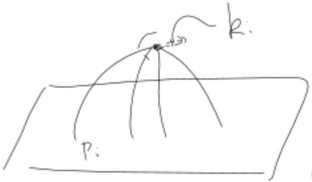
$$\rightsquigarrow \quad \lambda = -\frac{1}{2} \sum_i \Delta_i$$

$$\rightsquigarrow \quad \sum_i \underbrace{\Delta_i \left(\frac{P_i}{(P_i \cdot X)} + X \right)}_{\to\, k_i} = 0$$

$$k_j \cdot k_i \propto \Delta^2 \propto m^2$$

$$k_i = \Delta_i \left(\frac{p_i}{(p_i \cdot X)} + X \right) \qquad X \cdot k_i = 0$$

$$\sum_i k_i = 0$$



$\sigma_{ij} : 6$

$$\sigma_{ij} = - \frac{\Delta_i \Delta_j}{\sum_k \Delta_k} \frac{p_i \cdot p_j}{(p_i \cdot X)(p_j \cdot X)} \left[= \frac{\Delta_i \Delta_j}{\sum_k \Delta_k} \left(1 - k_i \cdot k_j \right) \right]$$

4

- $\sum_j \sigma_{ij} = \Delta_i$
- $\frac{\sigma_{ij}\sigma_{kl}}{\sigma_{ik}\sigma_{jl}} = \frac{(p_i \cdot p_j)(p_k \cdot p_l)}{(p_i \cdot p_k)(p_j \cdot p_l)} \simeq$ cross ratio

2

$n=4 \qquad \Delta_i = \Delta$

- **Conformal Mandelstam variables**

$$s = 4m^2\left(\frac{1-\sqrt{\rho\bar\rho}}{1+\sqrt{\rho\bar\rho}}\right)^2 \qquad t = 4m^2\left(\frac{\sqrt{\rho}+\sqrt{\bar\rho}}{1+\sqrt{\rho\bar\rho}}\right)^2$$

$(s+t+u=4)$ $$z = \frac{4\rho}{(1+\rho)^2}$$ $1+iT$

- Euclidean kinematics $\quad 0 \le s,t,u \le 4$

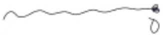
- **Amplitudes Conjecture**

$$\lim_{R\to\infty} \frac{\langle O_1 O_2 O_3 O_4\rangle - \text{GFF}}{\quad} = \hat{T}(s,t)$$

contact

ⓜ kinematics

• $\boxed{\rho, \bar{\rho}}$ $\boxed{s}$


$\longrightarrow$


• $\rho \to \tilde{\rho}\, e^{2\pi i}$ ⇝ $s = 4m^2 \left(\dfrac{1 + \sqrt{\tilde{\rho}\bar{\rho}}}{1 - \sqrt{\tilde{\rho}\bar{\rho}}} \right)^2 \geq 4m^2$

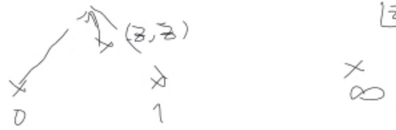
$\bar{\rho} \to \bar{\rho}$

• OPE limit $\rho, \bar{\rho} \to 0$ ⇝ $s \sim 4m^2,\ t, u \sim 0$

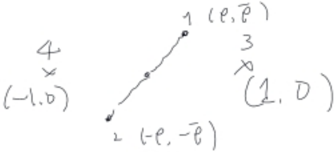
Regge limit $\longrightarrow$ Regge limit

Bulk pt limit $\rho = e^{-i\pi + \epsilon} e^{i\theta}$ $\bar{\rho} = e^{-i\pi - \epsilon} e^{-i\theta}$

$s \to \infty$ θ : finite



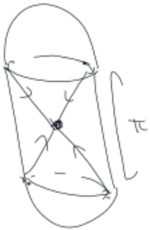
$\lfloor z, \bar{z}$

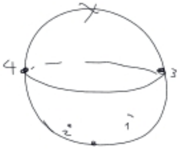


- Double Light cone limit

$$
\begin{cases}
s \sim 4m^2\left(1 - 2\sqrt{\varepsilon}\right) \\
t \sim 4m^2\left(1 - 2\sqrt{\eta}\right) \\
u \sim -4m^2\left(1 - 2\left(\sqrt{\varepsilon} + \sqrt{\eta}\right)\right)
\end{cases}
$$

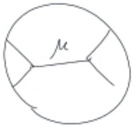
$$\varepsilon \ll \eta \ll 1$$





2. Dynamical subtlety

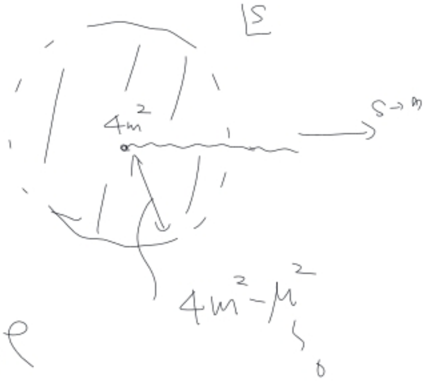
$\lim_{R\to\infty}$



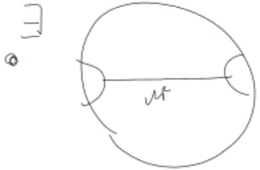
$\stackrel{?}{=} \dfrac{1}{s-\mu^2}$.

$= \dfrac{1}{s-\mu^2} \notin D$ "bad region"

$\infty \in D$



- Bad region is big for light exchange
- No bad region $\mu^2 > 4m^2$



other saddle