- Conformal Mandelstam variables

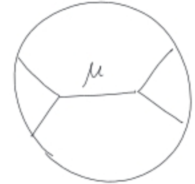
$$s = 4m^2\left(\frac{1-\sqrt{\rho\bar\rho}}{1+\sqrt{\rho\bar\rho}}\right)^2 \qquad t = 4m^2\left(\frac{\sqrt{\rho}-\sqrt{\bar\rho}}{1+\sqrt{\rho\bar\rho}}\right)^2.$$

- $\lim\limits_{R\to\infty}$ 

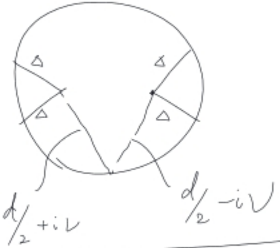
over 

$$= \begin{cases} \dfrac{1}{s-\mu^2} & \notin D \\ \infty & \in D \end{cases}$$

# Derivation




$$= \int dQ \int d\nu \, \frac{1}{\left(\nu^2 + \frac{d^2}{4}\right) - \mu^2 R^2}$$




$\nu \sim R$

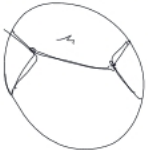
$$= \frac{1}{s - \mu^2}$$

+ (contribution from pole)

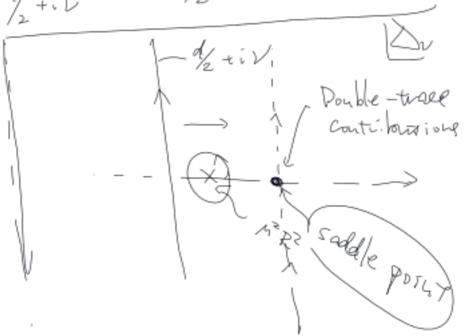
$-$ s-channel conformal block for $\Delta_\mu$

$(\mu^2 R^2 = \Delta_\mu(\Delta_\mu - d))$

$-$




$e^R$

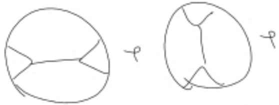


• Nonperturbative treatment

$$\langle O_1 \, O_2 \, O_3 \, O_4 \rangle = \sum_{O'} \underbrace{a_\Delta}_{\text{OPE coeff}} \; \underbrace{P_{\Delta,\ell}}_{\text{Polyakov-Regge block}}$$

Polyakov-Regge block

• s, t-channel symmetric conformal block

•  + contact terms

• $\iint dw\, d\bar{w}\; K(z,\bar{z}; w,\bar{w}) \times \mathrm{dDisc}\left[\text{conformal block}\right]$

$$\langle \overbrace{O_1 \, O_2 \, O_3 \, O_4}^{\text{identical}} \rangle_{\text{sub}}$$

$$= \sum_{O'} a_\Delta P_{\Delta,\ell} - \sum_{\Delta_{O'} < 2\Delta_\phi} a_{\Delta_{O'}} \left( \text{conformal}_{s+t} \right)$$

$$\Big\downarrow R \gg 1$$

Dispersive representation

$$T(s,t)$$

- Limit is finite (under certain assumption)

  Assumption: $E' = \{(s,t,u) \mid s,t,u \leq 2\}$

  $T(s,t)$ is finite (for fixed $u$)

  in the entire s-plane (1st sheet)

- Froissard-Martin bound

- (extended)(nonlinear) unitarity $(SS^{+} \leq 1)$

# "CFT amplitudes" and nonlinear unitarity


$$\text{"}|\text{in}\rangle\text{"} = \frac{e^{-iD\pi/2}\,|\psi\rangle}{\sqrt{\phantom{x}}}$$

$\sim$


$$\text{"}|\text{out}\rangle\text{"} = \frac{e^{+iD\pi/2}\,|\psi\rangle}{\sqrt{\phantom{x}}}$$

$$G = \begin{pmatrix} \langle \text{in}|\text{in}\rangle = 1 & \langle \text{out}|\text{in}\rangle \sim \text{"}S\text{"} \\ \langle \text{in}|\text{out}\rangle & \langle \text{out}|\text{out}\rangle = 1 \end{pmatrix}$$


$\langle \text{in}|\text{out}\rangle \sim S^*$

$G$: positive matrix $\Leftrightarrow$ unitarity

$1 - SS^\dagger \geq 0$

## Open Q

- $SS^\dagger = 1$ ?

$\downarrow$

-  $\rightsquigarrow$ Complete basis in CFT?

$O_{light}$ (below threshold) $\to$ orthonormal basis

$\sum_{O}$ 

# II Confinement & Mass gap in AdS

1210.5195 Aharony, Berkooz, Tong
Yankielowicz

Use AdS as IR regular.

$\rightsquigarrow$ Mass gap, confinement

# 1. YM in AdS & boundary conditions

$$S_{YM} = \frac{1}{g_{YM}^2}\int d^4x \sqrt{g}\,\frac{1}{4}\,\mathrm{tr}\left[F_{\mu\nu}F^{\mu\nu}\right]$$

sol. to free e.o.m

$$A_\mu(x,z) \sim a_i(x) + z^{d-2}\, g_{YM}^2\, J_i(x) + \dots$$


- 2 b.c. (preserving AdS isometry / well-defined variation)

Dirichlet: $a_i(x) \to 0 \rightsquigarrow \underline{J_i(x)}$

Neumann: $J_i(x) \to 0 \rightsquigarrow a_i(x)$

$F_{ij} = 0$

$F_{iz} = 0$

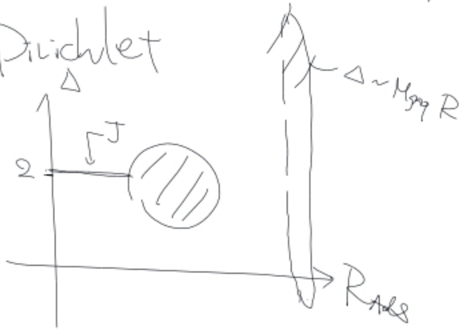
Dirichlet : 3d Conformal Th. with $SU(N)$ global sym.

Neumann : 3d Conformal Th. with $SU(N)$ gauge sym.

$\rightarrow$ "Perturbative" confinement

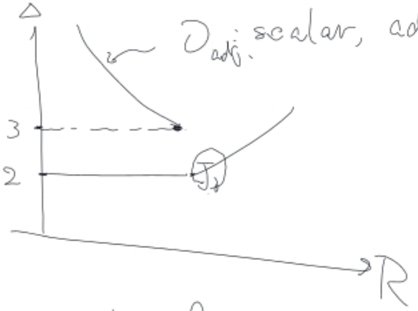
$\mathrm{tr}\left[F_{ij} F^{ij}\right]$ $\qquad \dfrac{dz^2 + dx^2}{z^2}$

(iii) Dirichlet




- D b.c. becomes unstable @ $R_{AdS}^{cr}$
- Mechanism for instability? [ABTY]

# I. Higgs (continuous)




$\mathcal{O}_{adj}$: scalar, adjoint SU(N))

$$\partial_i J^0 = \mathcal{O}_{adj}$$

$$\downarrow$$

$$J_i J^i\big|_{adj}$$

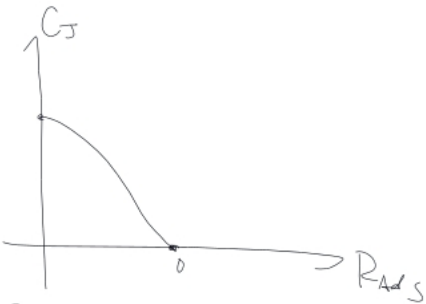
— : # of states may be too many ...?

$J_0$

## II. Decoupling (continuous?)

$\langle J\, J \rangle \sim C_J$

currents decouple
from other operators

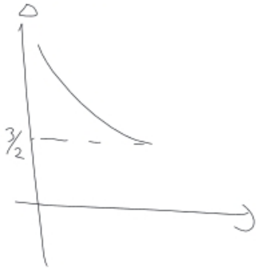



+ : Plausible

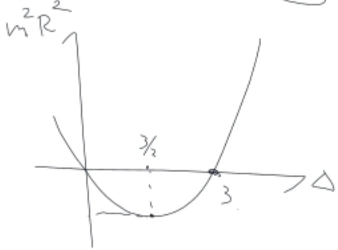
− : Post-decoupling b.c. is mysterious

## III Tachyon (discontinuous)

Some scalar op hits BF bound




$$\Delta = 3/2$$




$\Delta(\Delta - d)$, $d = 3$

$\rightarrow$ tachyonic instability ?

+ : This is what you learn in AdS/CFT

− : ? for interacting theory