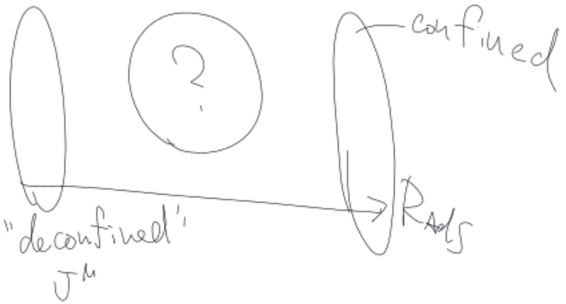
I Higgs $\quad \partial_\mu J^\mu = O_{adj}$ $\quad$ too many states..?

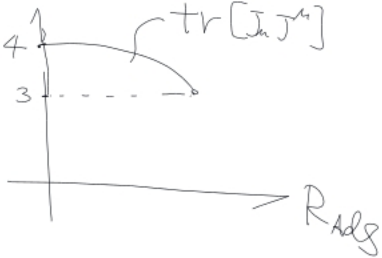
II. Decoupling $\quad \langle J\, J \rangle \sim C_J \to 0$

III. Tachyon $\quad \Delta_{O_{scalar}} \to 3/2$ $\quad$ BF bound, $\quad$ perturbative



IV. Marginal Singlet [Cooper: Di Pietro, Ju, SK]

A scalar singlet (under $SU(N)$) $\mathcal{O}_{singlet}$ becomes marginal, triggering the RG flow

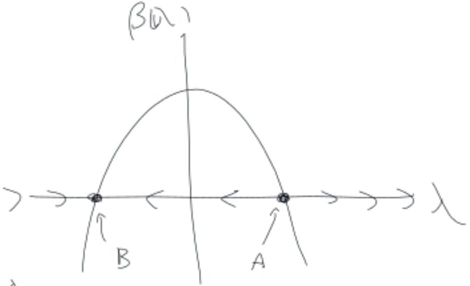


- In weakly coupled theory,

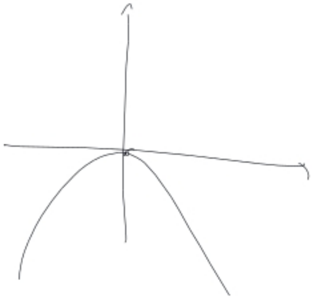
  Tachyon $\longrightarrow$ Double trace op

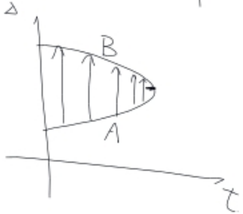
  $\Delta = 3/2$ $\quad\quad$ $\Delta = 3$

- Often coming from fixed point merger and annihilation



$\longrightarrow$





- Fixed point merger
- 2309.10031 [Lauria, Milam, van Rees]

Setup QFT in AdS with $t \int \mathcal{O}(x,z)$

$$\mathcal{O}(x,z) = z^{\hat{\Delta}(t)} \underbrace{\hat{\mathcal{O}}_{bdy}}_{\text{leading}} \Big|_{t = t_c + \delta t}$$

$$\hat{\Delta}(t_c) = d$$

$t = t_c + \delta t$

$$+ \delta t \int d^{d+1}x \, \mathcal{O}(x, z)$$

$$\underbrace{+ \, n \log\left(\epsilon / R_{AdS}\right) \int d^{d}x \, \mathcal{O}(x, \epsilon)}_{\text{finiteness of correl.}}$$

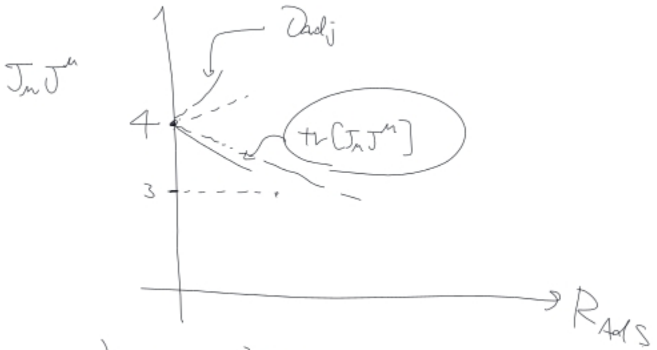
$$+ \hat{g} \int d^{d}x \, \mathcal{O}(x, \epsilon)$$

$\hat{g}$ needed for $\begin{cases} \text{conf. sym} \\ \text{Ward identity} \end{cases}$

$$\delta t - \underbrace{(\;) \, \hat{g}^2}_{\text{definite sign}} = 0$$

3. Perturbative test of YM in AdS_4

~~Higgs~~

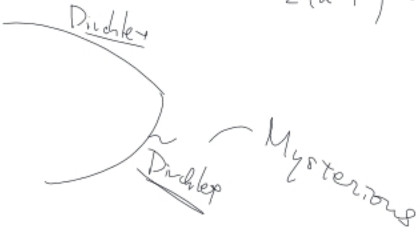


- $\lambda_{crit} = g_{YM}^{2\,crit} N \sim 21.5 \rightsquigarrow (R_{AdS}\Lambda_{YM}) \sim 0.37$
- $C_J = 0$? at $\lambda^{C_J}_{crit} \rightsquigarrow \lambda^{C_J}_{crit} \sim 272$

Marginal singlet shows up earlier than decoupling

$$\underline{\underline{\Delta_D}} \doteq d+1 - \frac{4B(a)}{d}\frac{C_{DD}(a)}{C_{TD}(a)}\Delta_D$$

$$2(d-1) \sim d.$$



III. Modified Crossing & Non-inv. Symmetry

- In theories with non-inv. symmetry

- Integrable flows in 1+1d
  gapped phase
- Difference of "basis" in $\left|\begin{array}{l}\text{flat space}\\ \text{AdS} \sim \text{CFT}\end{array}\right.$

Non inv. Symmetry

- Symmetry in QFT $\subset$ Topological op.

$$\sim \exp\left[\alpha \int * j\right] =$$

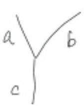
$$d * j = 0$$

- Usual sym. $g_a \in G$ $\qquad g_a \cdot g_b = g_c \qquad g_{a,b,c} \in G$

- Non-inv. (fusion category)

$$\mathcal{L}_a \cdot \mathcal{L}_b = \sum_c N_{ab}^{c} \, \mathcal{L}_c \qquad N_{ab}^{c} \in \mathbb{Z}_{\geq 0}$$

1+1d

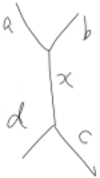
- Data
  - Fusion coeff N_{ab}^{c} (if $N_{ab}^{c} \neq 0$ $\exists$ 

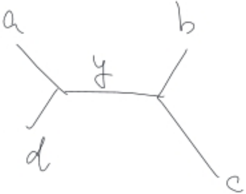
)
  - Quantum dim

$$\langle L_a \rangle =$$ 

$$= d_a$$

  - F-symbol



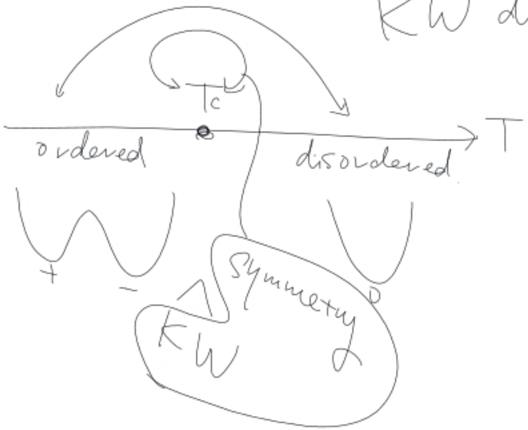
$$= \sum_y F^{abc}_{d\; xy}$$ 

Consistency condition

$$d_a \cdot d_b = \sum_c N_{ab}^{c} d_c$$

Example of Th with non-inv.

- 2d Ising @ critical point



KW duality

KW

$|+\rangle \to |0\rangle$

$|-\rangle \to |0\rangle$

$|+\rangle + |-\rangle \leftarrow |0\rangle$

• Symmetry of 2d Ising CFT $(1, Y, N)$

$I_{\mathbb{Z}_2}$

$$Y^2 = 1 \qquad \underset{KW}{\underline{N}}^2 = 1 + Y \qquad NY = YN = N$$

$$|+\rangle \xrightarrow{N} |0\rangle$$
$$|0\rangle \xrightarrow{N} |+\rangle + |-\rangle$$

• All relevant def breaks N

• Tritical Ising CFT

$-\phi_{13} \left(\Delta = \frac{6}{5}\right)$

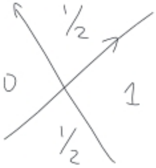
preserve $1, Y, N$

3 vacua

$$N|0\rangle = |+\rangle + |-\rangle$$
$$N|+\rangle = N|-\rangle = |0\rangle$$

- RG flow preserves integrable

- S-matrix : S-matrix of kinks btwn neighboring vacua



(S)

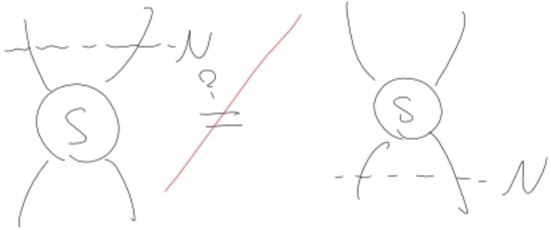
$s = 4m^2 \cosh^2\frac{\theta}{2}$ ← rapidity parametrization

$$S^{ab}_{dc}(\theta) = Z(\theta) \underbrace{\left(\frac{d_a d_c}{d_b d_d}\right)^{\frac{i\theta}{2\pi}}}_{\text{factor}} \left[\sqrt{\frac{d_a d_c}{d_b d_d}} \sinh\left(\frac{\theta}{3}\right) \delta_{bd} + \sinh\left(\frac{i\pi - \theta}{3}\right) \delta_{ac} \right]$$

- Unitarity
- Crossing
- YB

$d_0 = d_1 = 1$

$d_{1/2} = \sqrt{2}$

- 

$[N, \hat{S}] = 0$

- $S^{\text{new}} = S^{\text{Zam}} / \text{factor}$ ⇝ non-inv ✓

⇝ $S^{\text{new}}{}^{ab}_{dc}(\theta) = \underbrace{\sqrt{\frac{d_a d_c}{d_b d_d}}}\, S_{\text{new}}{}^{bc}_{ad}(i\pi - \theta)$

Modified crossing!

Derivation of Modified crossing

- Key inputs

  - IR is gapped & TQFT with $1, N, Y$

  - Vacua are in 1-1 correspondence with sym lines
    3 vacua

    $$|0\rangle \longleftrightarrow \mathbb{1}, \quad |\tfrac{1}{2}\rangle \longleftrightarrow N, \quad |1\rangle \longleftrightarrow Y$$

    $$N|\tfrac{1}{2}\rangle = |0\rangle + |1\rangle$$

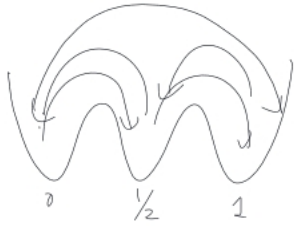
    $$N \cdot N = 1 + Y$$

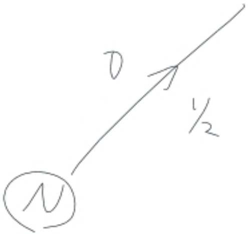
  - All vacua can be obtained from $|0\rangle$

    $$|\tfrac{1}{2}\rangle = N|0\rangle$$

    $$|1\rangle = Y|0\rangle$$

• World line of particles $\simeq$ symmetry line $\mathcal{N}$

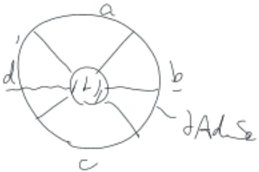




Config. kinks

$\simeq$ network of lines

$$\mathcal{N}|0\rangle = |1/2\rangle$$
$$\mathcal{N}|1/2\rangle = |0\rangle + |1\rangle$$

• $S^{ab}_{dc}(\theta) \propto$ 

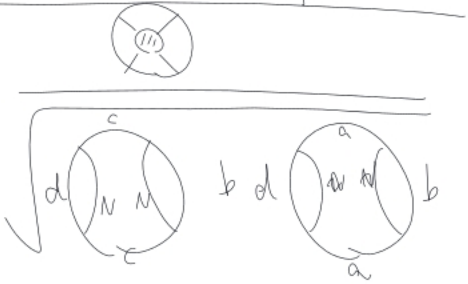
flat-space limit

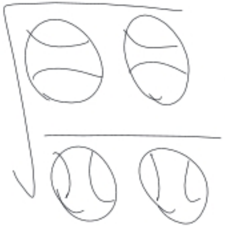
• in/out states are not normalized

$|in\rangle =$ 

$\langle out| =$ 

$\langle in|in\rangle \propto$ 

$S^{ab}_{dc} =$ 

$S(\theta) =$  $S(i\pi-\theta)$