Bootstrapping 2d loop models

$$\tilde{S} = \{V^d_{\langle 1,s\rangle}\}_{s\in\mathbb{N}^*} \cup \{V_{(r,s)}\}_{\substack{r\in\frac{1}{2}\mathbb{N}^* \\ s\in\frac{1}{2r}\mathbb{Z}}} \cup \{V_P\}_{P\in\mathbb{C}}$$

O(n) model	U(n) model	Potts model (S_Q)
$\{V^d_{\langle 1,s\rangle}\}_{s\in 2\mathbb{N}+1} \cup \{V_{(r,s)}\}_{\substack{r\in\frac{1}{2}\mathbb{N}^* \\ s\in\frac{1}{r}\mathbb{Z}}}$	$\{V^d_{\langle 1,s\rangle}\}_{s\in\mathbb{N}^*} \cup \{V_{(r,s)}\}_{\substack{r\in\mathbb{N}^* \\ s\in\frac{1}{r}\mathbb{Z}}}$	$\{V^d_{\langle 1,s\rangle}\}_{s\in\mathbb{N}^*} \cup \{V_{(r,s)}\}_{\substack{r\in\mathbb{N}+2 \\ s\in\frac{1}{r}\mathbb{Z}}} \cup \{V_{P_{(0,s)}}\}_{s\in\mathbb{N}+\frac{1}{2}}$

$$\langle V_{(2,0)} V_{(2,0)} V_{(2,0)} V_{(2,0)} \rangle$$

$1, \sigma, \varepsilon$

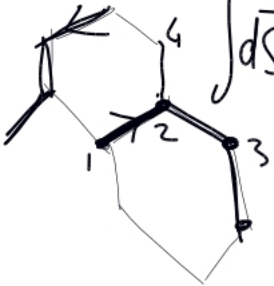
$$\langle V_{(\frac{1}{2},0)} V_{(\frac{1}{2},0)} V_{(2,0)} V_{P_{(0,\frac{1}{2})}} \rangle$$

$$\langle V_{(\frac{3}{2},0)} V_{(\frac{1}{2},0)} V_{P_1} V_{P_2} \rangle \qquad P_1, P_2 \in \mathbb{C}$$

O(n) model. $E = \sum_{\langle i,j\rangle} \log\left(1 + K\, \vec{S}_i \cdot \vec{S}_j\right)$

$\vec{S}_i \in \mathbb{R}^n$

$$Z = \int \prod_{\langle i,j\rangle} \left(1 + K\, \vec{S}_i \cdot \vec{S}_j\right)$$

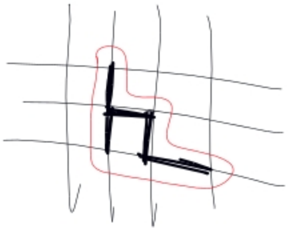


$$\int d\vec{S}_2 \, (\vec{S}_1 \cdot \vec{S}_2)(\vec{S}_2 \cdot \vec{S}_3) \cancel{(\vec{S}_2 \cdot \vec{S}_4)}$$

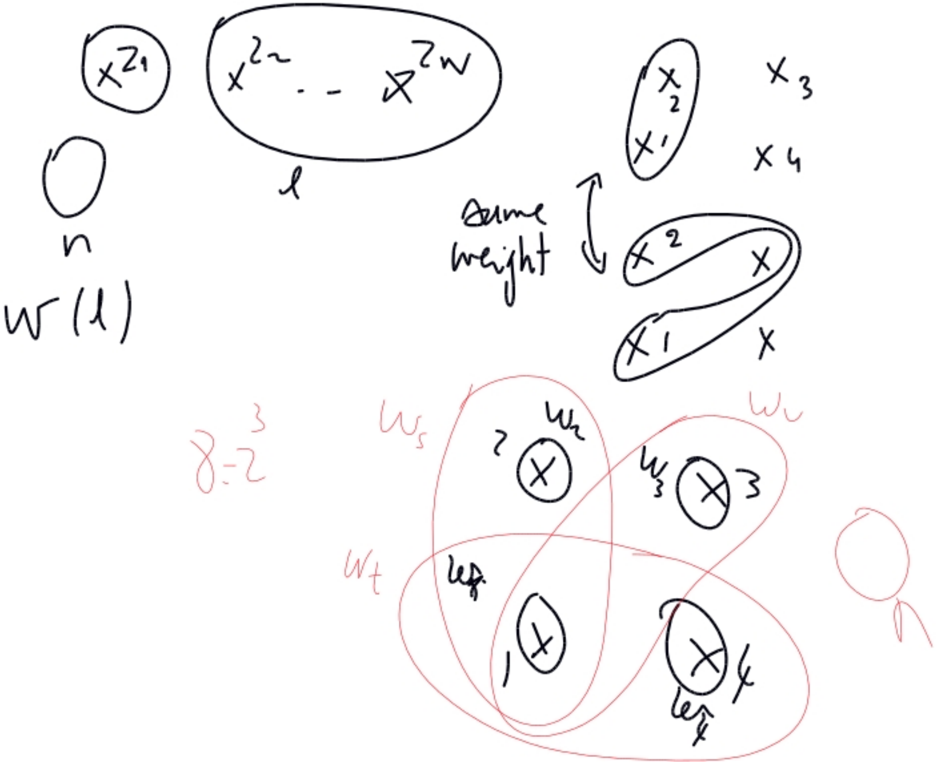
$$Z = \sum_{\text{config. of loops}} K^{\#\text{occupied edges}} \; n^{\#\text{loops}}$$

$$\mathrm{Tr}_{\mathbb{R}^n} 1 = n$$

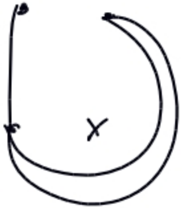
$$K = K_c(n) = \frac{1}{\sqrt{2 - \sqrt{2-n}}} \qquad n \in \emptyset$$

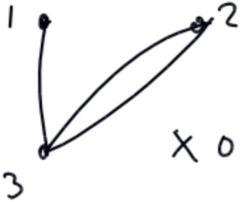


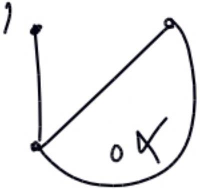
$\sigma \in \{1 \dots e\}$



valency 3

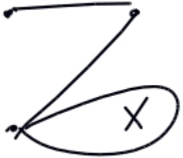


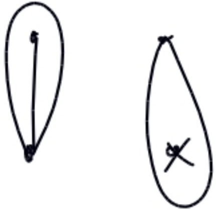




valency $2r$, $r \in \frac{1}{2}\mathbb{N}^*$

$\sum r \in \mathbb{N}$





$$\Psi_{(r,s)} = e^{\frac{i}{2} s \sum_{i=1}^{2r} \theta_i}$$

$$Z = \sum_{\text{corp entry}}$$

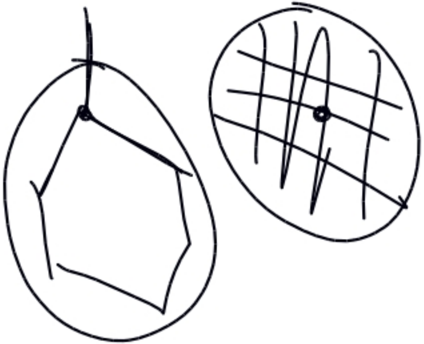
$$\langle V_1 V_2 V_3 V_4 \rangle_M$$

$$\langle V^{i_1}_{(\frac{1}{2},0)} V^{i_2}_{(1/2,0)} V^{i_3}_{(1/2,0)} V^{i_4}_{(1/2,0)} \rangle$$

$$(v_1 \cdot v_2)(v_3 \cdot v_4)$$
$$(v_1 \cdot v_4)(v_2 \cdot v_3)$$
$$(v_1 \cdot v_3)(v_2 \cdot v_4)$$

$$Z = \sum_{\text{loop config}} \prod_{l \text{ closed loop}} w(l) \prod_{i=1}^{N} W_{(r,s)}(\text{angles})$$

$$\lim_{\text{critical}} Z = \left\langle \prod_{i=1}^{N} V_{(r_i,s_i)}(z_i) \right\rangle$$



$$n \in (-2,2)$$

$$V_p = V_{(0,2p)}$$

$$P_{(1,3)}$$

Kac indices r, s : $\begin{cases} r \in \frac{1}{2}\mathbb{N}^* \\ rs \in \mathbb{Z} \end{cases}$ $\quad \sum r \in \mathbb{Z}$

$V_{(r,s)}$

Primitive r, s : $\begin{cases} r \in \frac{1}{2}\mathbb{N}^* \\ \underline{rs \ \text{spin}} \in \mathbb{Z} \end{cases}$ $\quad \sum r \in \mathbb{Z}$

$$w_{(r,s)} = e^{\frac{i}{2} s \sum^{2r} \theta_i}$$

$$\theta_i \to \theta_i + \delta\theta$$

$$w_{(r,s)} \to w_{(r,s)} e^{irs\,\delta\theta}$$

$$V_P = V_{(0, 2\beta P)}$$

$$w(P) = e^{\frac{i}{2}(2\beta P)2\pi} + e^{-\frac{i}{2}(2\beta P)2\pi}$$
$$= 2\cos(2\pi\beta P)$$

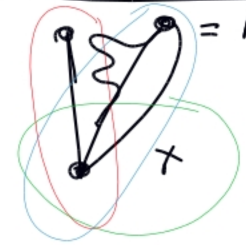
$$w(p) = 2\cos(2\pi\beta p)$$

$V^d_{(1,n)}$

$$n = w(P_{(1,1)}) = -2\cos(\pi\beta^2)$$

$$c = 13 - 6\beta^2 - 6\beta^{-2}$$

$\sigma = (1,1,1)$



$= M$

$$\left\langle V_{(3/2,0)}\, V_{(1/2,0)}\, V_{(1,1)}\, V_P \right\rangle_M$$

$\mathcal{S}^{(u)} = \ldots$

$\sigma = (1, 3/2, 1/2)$

$\sigma^{(s)} = 1$

$\sigma^{(t)} = 3/2$

$\sigma^{(u)} = 1/2$

$$\mathcal{S}^{(s)} = \{V_{(r,s)}\}_{r \in \mathbb{N}^*,\ s \in \frac{1}{r}\mathbb{Z}}$$

$$\mathcal{S}^{(t)} = \{V_{(r,s)}\}_{r \in \frac{3}{2}+\mathbb{N},\ s \in \frac{1}{r}\mathbb{Z}}$$

$$\langle \ \rangle_M \longrightarrow \left(D_k^{(x)} \right)_{k,x} \qquad C^{ref} = \prod \Gamma_\beta^{-1}$$

$$\frac{D_k^{(w)}}{D_k^{ref\,(w)}} = d_k^{(s)}(P_s) \qquad D_k^{(s)ref} = \frac{C^{ref}_{12k}\, C^{ref}_{34k}}{B_k^{ref}}$$

$$d_k^{(s)}(P_s) = d_k^{(s)}(P_s + \beta^{-1})$$

w_s

$\sum \prod$ weights

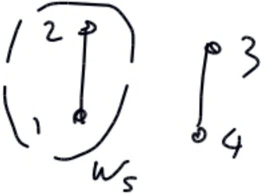
$$w_s = 2\cos(2\pi\beta P_s)$$

$\rightarrow V_{P_s}$

$$\Longrightarrow d_k^{(s)}(w_s, n) \qquad n = -2\cos(\pi\beta^2)$$

$\rightarrow$ poly.

$$\left\langle V^{4}_{(\frac{1}{2},0)} \right\rangle$$



$$S^{(s)} = \{V_{(r,s)}\}_{\substack{r\in\mathbb{N}^* \\ s\in\frac{1}{2}\mathbb{Z}}} \cup \{V_{P_s + j\beta^{-1}}\}_{j\in\mathbb{Z}}$$

$$d^{(s)}_{\text{diag.}} = 1$$

$$d^{(s)}_{(1,0)} = 0$$

$$d^{(s)}_{(2,0)} = n^2-4$$

$$d^{(s)}_{(2,1)} = -(n^2-4)$$

$$\boxed{\begin{aligned} d^{(s)}_{(4,0)} = {} & n^2(n-2)^3(n+1)^2(n+2) \\ & \times \Big[w_s(n+2)^2(n-1)^2 \\ & \qquad + 2n^4-6n^2-8n+16\Big] \end{aligned}}$$

$$S^{(A)} = \{V_{(r,s)}\}_{\substack{r\in\mathbb{N}^* \\ s\in\frac{1}{2}\mathbb{Z}}}$$

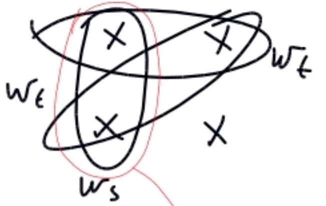
$$3\,d^{(s)}_{(3,0)} = -8n^2(n-2)^2(n+2)$$

$$3\,d^{(s)}_{(3,\frac{2}{3})} = 4(n^2-1)(n^2-3)(n-2)$$

$$\cos\pi\left(\tfrac{p}{q}\right)$$

$$\omega_{(r,s)} = e^{\frac{i\pi}{2}\varepsilon\theta}$$

$$\left\langle V^{(2,1)}_{P_1} V_{P_2} V_{P_3} V_{P_4} \right\rangle_M$$



defect of para w_s

$$P = P_{(0,1/2)} \in \mathrm{Potts}$$

$$w(P_{(0,1/2)}) = 0$$

fct of $n, w_s, w_t, w_u, w_1, w_2, w_3, w_4$

$$d^{(s,t,u)}_{\text{diag}} = 1$$

$$d^{(s)}_{(1,0)} = w_t + w_u$$

$$d^{(s)}_{(1,1)} = w_u - w_t$$

$$d^{(s)}_{(2,0)} = (n^2-4)(w_t^2 + w_u^2 + 2w_s - 4) - (n-2)(w_t + w_u)(w_1 + w_2)(w_3 + w_4) - (n+2)(w_t - w_u)(w_1 - w_2)(w_4 - w_3)$$

Solve model?

- compute all struct csts.
- all maps V
- factorize 4pt $\to$ 3pt

18 maps

$V_{(2,0)}$

$[4] + [22] + [211] + [1] + [1]$

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