Statistics of 3D Black Holes from Liouville Line Defects

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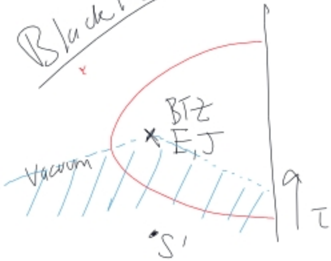
Thin shell black holes and wormholes = Statistics of line defects D_Σ in dual CFT

Line defect L_Σ in Liouville

Gravity

$$-\int \sqrt{g}\,(R+2) \qquad + \; m\int d\varphi\, d\ell$$

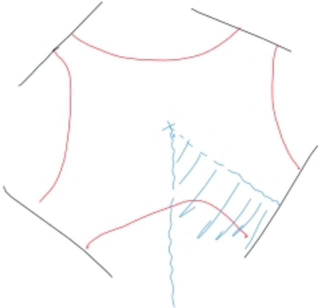
Black Hole



$$= \langle D_\Sigma^\dagger D_{\Sigma'} \rangle$$

$$D_\Sigma|0\rangle = |\psi\rangle$$

Wormholes



$$= \frac{\qquad}{\langle D_\Sigma^\dagger D_\Sigma \rangle^4}$$

??

Q: How to reproduce this in dual CFT?

"Identity Block" $\rightsquigarrow$ Cardy/Tauberian theorem

$\rightsquigarrow$ reinterpret as ensemble

Liouville

$$S_L = \int \partial\phi\bar{\partial}\phi + e^{\phi} \qquad V_\alpha = e^{2\alpha\phi}$$

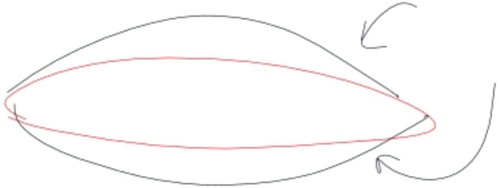
$$L_\Sigma = e^{\frac{m}{4\pi}\int_\Sigma \phi}$$

Goal #1: find $\langle P | L_\Sigma | P' \rangle$

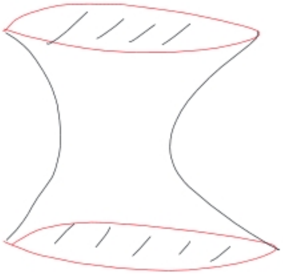
$c \to \infty$: classical solutions

$$\partial \bar{\partial} \phi = \frac{1}{2} e^{\phi} \qquad \Delta[\partial_n \phi] = -2m$$

$$e^{\phi} |dz|^2$$

$\langle L_\Sigma \rangle_{\text{sphere}} =$  $e^{\phi} = \dfrac{1}{\sinh^2(y \pm A)}$

$$\langle L_\Sigma^{(t)\dagger} L_\Sigma^{(s)} \rangle_{\text{sphere}} =$$



$$\approx e^{-\frac{c}{6} S_L^{\text{on-shell}}}$$

$$\hookrightarrow \langle 0 | L_\Sigma | P \rangle$$

$$\langle L_\Sigma^\dagger L_\Sigma \rangle_{\text{torus}}$$

$$\hookrightarrow \langle P' | L_\Sigma | P \rangle = e^{\frac{c}{6} \boxed{C_L(h,h')}}$$

known.

<u>On disk:</u>

ZZ boundary condition

$$e^{\phi} \sim \frac{4}{-(z-\bar{z})^2}$$

$$\langle 0 | L_\Sigma | ZZ \rangle$$

$$= \int dP \, \underbrace{\langle 0 | L_\Sigma | P \rangle\!\rangle} \, \underbrace{\langle\!\langle P | ZZ \rangle}_{\text{known.}}$$

Dual CFT_2

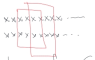
any $c > 1$

* line defect D_Σ that transforms like L_Σ (m)
* $D^\dagger_\Sigma D_\Sigma \supset \mathbb{1}$

Claim: universal matrix elements @ high E

$$\boxed{\overline{|\langle i|D^{(m)}_\Sigma|j\rangle|^2}^{\text{states}} \approx \left(C_0^D(h_i, h_j; m)\, C_0^D(\bar{h}_i, \bar{h}_j; m) \right)}$$

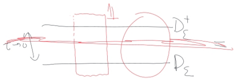
Ex @ large c

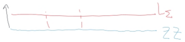


$$\lim_{n\to\infty} \prod_{i=1}^{n} \mathcal{O}(z_i)$$

$$e^{\frac{m}{4\pi}\int\varphi} = \lim \prod V_\alpha$$

Derivation of $C_0^D(0,h;m)$





$$\mathcal{F}_{\mathbb{1}}^{D} = N \langle 0 | L_\Sigma | ZZ \rangle \qquad \text{(exact)}$$

universal matrix elements

$$C_0^D(h,h') \sim \langle h' | L_\Sigma | h \rangle \, e^{-\frac{1}{2}\left(S(h)+S(h')\right)}$$

Cardy

$$L_\Sigma^\dagger L_\Sigma \sim |ZZ\rangle\langle ZZ|$$

$$D_\Sigma^\dagger D_\Sigma$$

Back to Gravity

$D_\Sigma \longleftrightarrow$ dust shell

<u>Claim</u>: dual to an approx. Gaussian ensemble of $\langle i | D_\Sigma | j \rangle$ with variance $|C_0^D|^2$

Black hole

$$d\rho^2 + \cosh^2\rho\, e^{\phi} |dz|^2$$

soliton

$$\langle D_\Sigma^\dagger D_\Sigma \rangle = Z_{grav} = e^{-2S_L}$$

$$= \left| \langle 0 | L_\Sigma | ZZ \rangle \right|^2$$

$$= \langle D_\Sigma^\dagger D_\Sigma \rangle \Big|_{\mathbb{1}}$$

Wormholes

expand

$$|\psi\rangle = D_\Sigma^\dagger |0\rangle = \sum_p d_p |p\rangle$$

Then

$$Z_{wormhole} = e^{-I_{grav}} = \overline{\langle\psi|\psi\rangle\langle\psi|\psi\rangle\langle\psi|\psi\rangle}$$

with $\overline{d_p d_q} = \delta_{pq}\, C_0^D(0,h_p)\, C_0^D(0,\bar{h}_p)$

Comments

* higher D

* In UV-complete theories?

$$Z(\tau,\bar{\tau})\,Z(\tau,\bar{\tau})$$

Higher topologies → destabilized?
→ canceled?

UV (no wormholes)

time average?

state average?

RG (Maldacena-Qi) (Almheiri-Lin)

large-N jitter

EFT (yes wormholes)