Where is tree-level string theory?

2406.12959

with Jan Albert & Leonardo Rastelli

TREE-LEVEL : $2 \to 2$ scattering of gravitons in max sugra

- Analyticity

10,d

$$\langle \phi\phi\phi\phi \rangle \sim s^4 M(s,u)$$

- Crossing-symmetry : $s \leftrightarrow t \leftrightarrow u$

- Unitarity : $\operatorname{Im} M(s,u) = s^3 \sum_{J\,\text{even}} n_J\, \rho_J(s)\, P_J\left(1+\frac{2u}{s}\right)$

$$\uparrow$$

$$0 < \rho_J$$

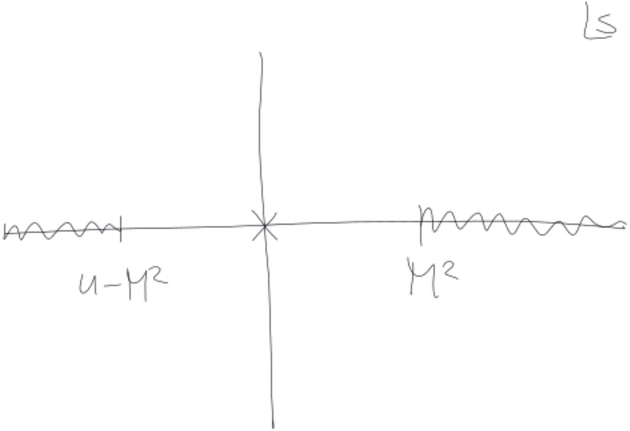
- Regge behavior : $|s| \to \infty$, $u<0$

$$s^2 M(s,u) \to 0$$

Setup:

IR: $$M_{IR}(s,u) = \frac{8\pi G_N}{stu} + g_0 + g_2(s^2+t^2+u^2) + \ldots$$

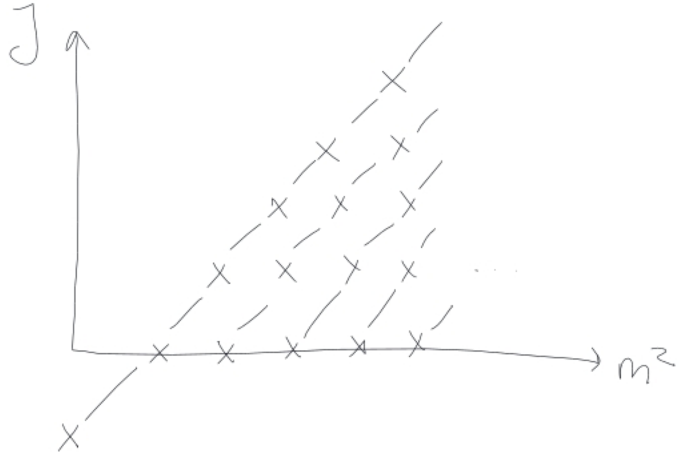
UV: cutoff M,



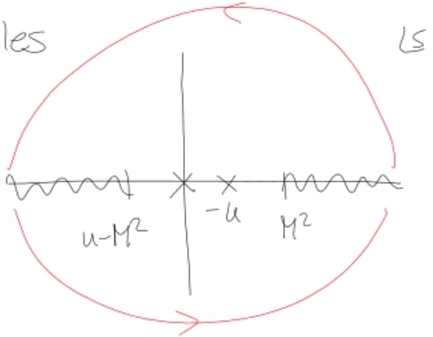
Virasoro-Shapiro amplitude:

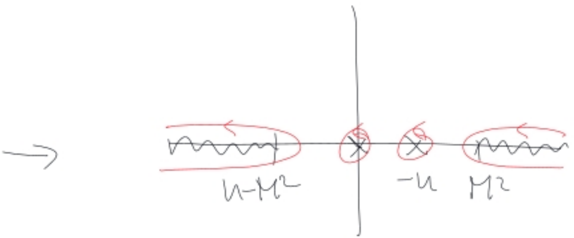
$$M_{ST} = \frac{\Gamma(-\alpha' s)\Gamma(-\alpha' t)\Gamma(-\alpha' u)}{\Gamma(1+\alpha' s)\Gamma(1+\alpha' t)\Gamma(1+\alpha' u)}$$

$$M_{ST}(s,u) \sim s^{-2+\alpha' u}$$



Sum rules





$$\oint \frac{ds}{s} \frac{M(s,u)}{[s(s+u)]^{k/2}} = 0$$

$k \geq -2$

$$\operatorname*{Res}_{s=0,-u}\left[\frac{1}{s}\frac{M(s,u)}{[s(s+u)]^{k/2}}\right] =$$

$$= \# \int_{M^2}^{\infty} dm^2 \left(\frac{1}{m^2} + \frac{1}{m^2+u}\right) \operatorname{Im}\left[\frac{M(s,u)}{[s(s+u)]^{k/2}}\right]$$

$$\equiv \left\langle \frac{2m^2+u}{m^2+u} \frac{P_J\left(1+\frac{2u}{m^2}\right)}{[m^2(m^2+u)]^{k/2}} \right\rangle$$

$k=-2$ $$-\frac{8\pi G_N}{u} = \langle m^2(2m^2+u) P_J(1+\frac{2u}{m^2})\rangle$$

$k=0$ $$g_0 + 2g_2 u + \dots = \left\langle \frac{(2m^2+u)\, P_J(1+\frac{2u}{m^2})}{m^2+u} \right\rangle$$

– $g_0 = \langle 2 \rangle$, $g_2 = \langle \frac{1}{m^4} \rangle$

$$-\int_0^{M^2} \frac{8\pi G_N}{u} f(u) = \left\langle \int \dots f(u) \right\rangle$$

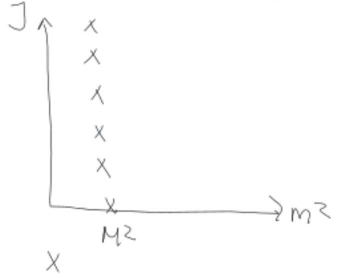
$$\tilde{g}_0 = \frac{g_0}{8\pi G_N} M^6$$

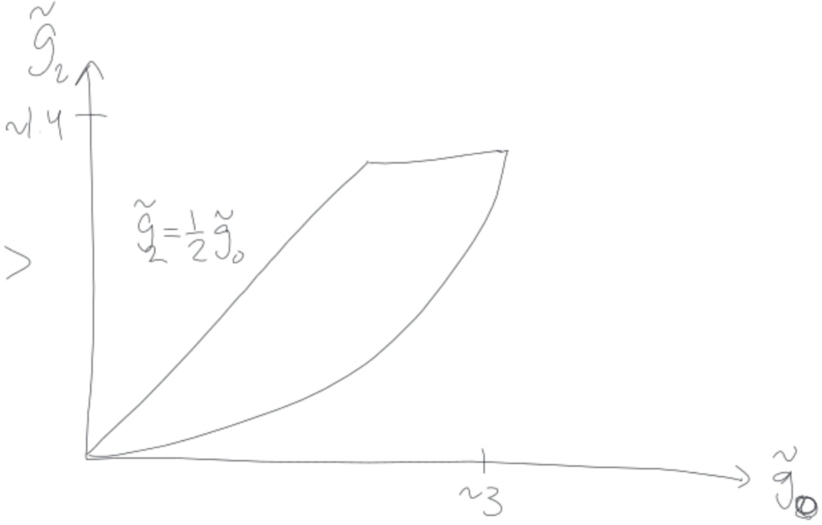
Null constraints

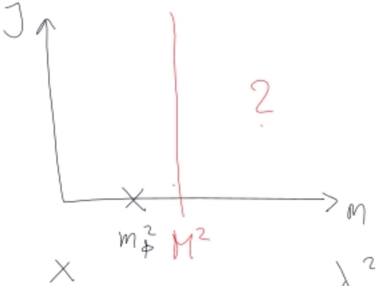
$$\left\langle \frac{2J(7+J)-4}{m^2} \right\rangle = 0$$

bound $\tilde{g}_0$ vs $\tilde{g}_2$

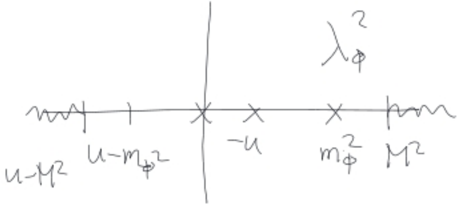
$g_0 = \langle 2 \rangle$, $g_2 = \langle \frac{1}{m^4} \rangle$



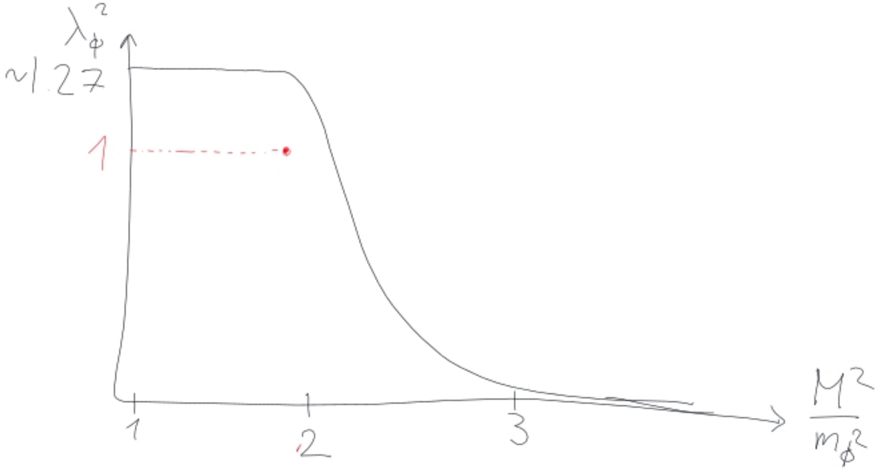




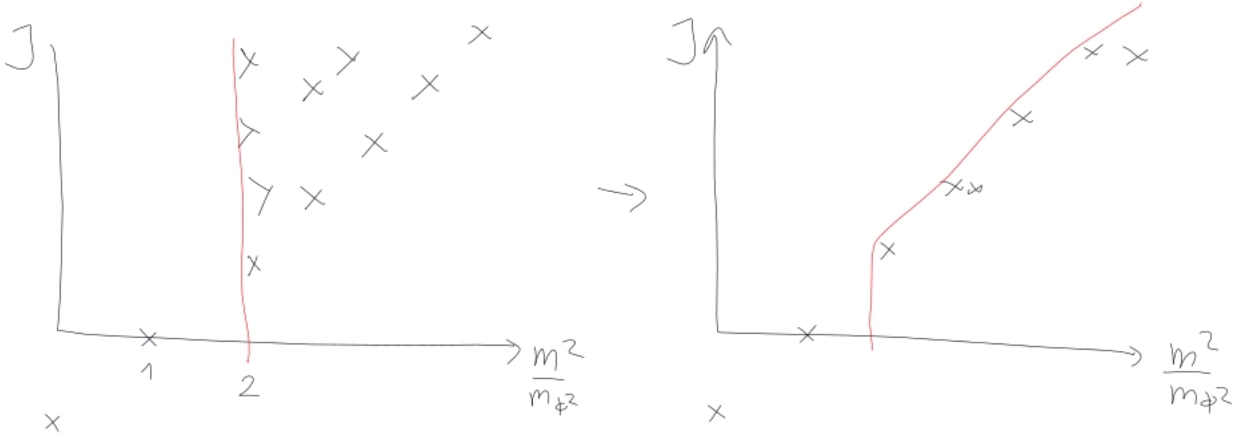
J
m
m_ϕ^2
M^2
2
X



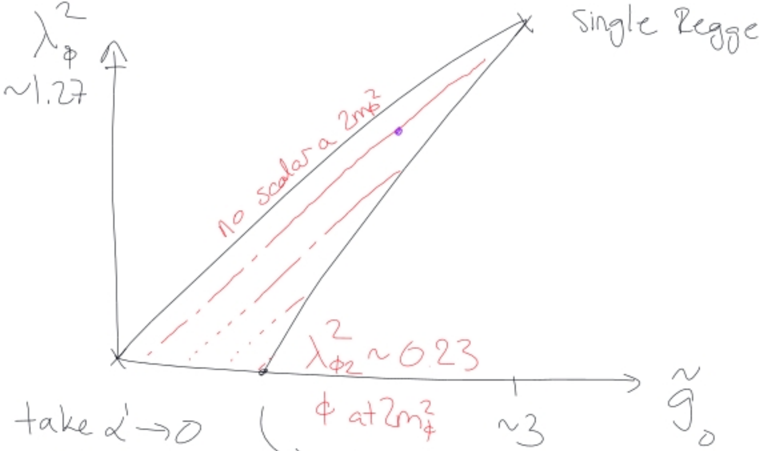
λ_ϕ^2
$u-M^2$
$u-m_\phi^2$
$-u$
m_ϕ^2
M^2



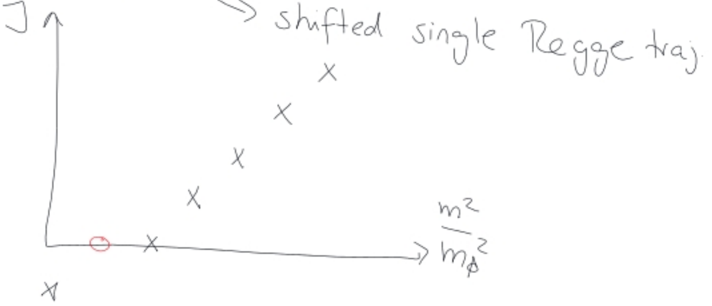
λ_ϕ^2
~1.27
1
1
2
3
$\frac{M^2}{m_\phi^2}$



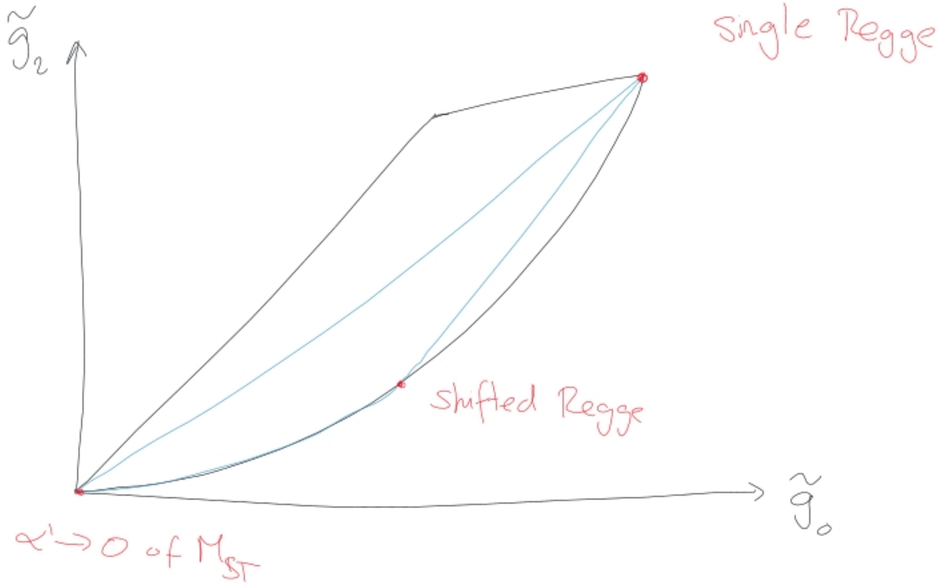
$\lambda_\phi^2 \sim 1.27$ $\lambda_\phi^2 \sim 1.27$



take $\alpha' \to 0$



$$\frac{M^2}{m_\phi^2} = 2$$



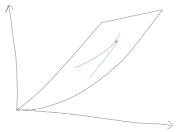
$\tilde{g}_2$
Single Regge
Shifted Regge
$\tilde{g}_0$
$\alpha' \to 0$ of M_{ST}

Nima:

$$M_{VS}(s,u) - \varepsilon \frac{\Gamma(1-\alpha' s)\Gamma(1-\alpha' t)\Gamma(1-\alpha' u)}{\Gamma(2+\alpha' s)\Gamma(2+\alpha' t)\Gamma(2+\alpha' u)}$$

$0 \le \varepsilon \le 1$

$-\frac{3}{16}$



$$M_{4F3}(s,u) = M_{VS}(s,u)\, {}_4F_3\left(\begin{matrix} 1-\alpha' s, -\alpha' t, \alpha' u, x \\ 1+\alpha' s, 1+\alpha' t, 1+\alpha' u \end{matrix} ; 1 \right) \qquad 0 \le x \le 1$$