- First-order transition
- Complex fixed point
- Complex CFT
- A microscopic model.

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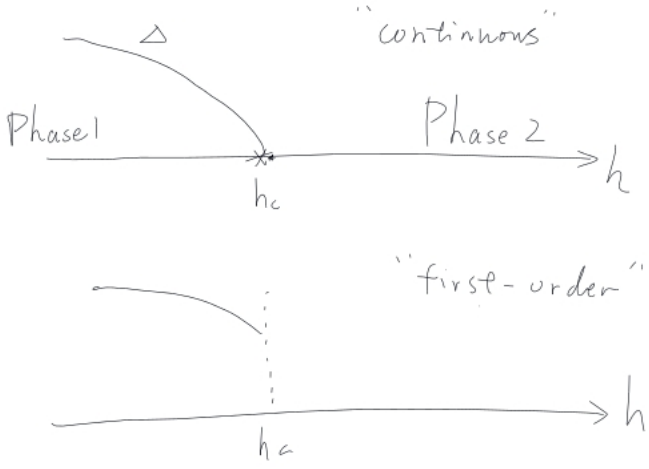
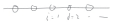
* Yin Tang, Q. Tang, H. Ma, Yin-chen He, WZ.
  arXiv. 2403. 00852

Phase transition



Q-state Potts model

Potts spin Q=2 Q=3 Q=4, 5, ···



$$H_0(J,h) = -\sum_{i=1}^{L}\sum_{k=1}^{Q-1}\left(J\left(\sigma_i^{+}\sigma_{i+1}\right)^k + h\,\tau_i^k\right)$$

phase op $\sigma|n\rangle = \omega^n|n\rangle$ $n = 0, 1, \cdots, Q-1$, $\omega = e^{i\frac{2\pi}{Q}}$

shift op $\tau|n\rangle = |(n+1) \bmod Q\rangle$

$$\sigma = \begin{pmatrix} 1 & & & \\ & \omega & & \\ & & \ddots & \\ & & & \omega^{Q-1} \end{pmatrix} \qquad \tau = \begin{pmatrix} 0 & 1 & & \\ & 0 & \ddots & \\ & & \ddots & 1 \\ 1 & & & 0 \end{pmatrix}$$

$$\sigma\tau = \tau\sigma\cdot\omega.$$

1) spin permutation sym.

eg $Q=3$. $\begin{cases} \text{cyclic sp} & S=\prod_{j=1}^{L}\tau_j \\ \text{charge conj} & C \end{cases}$

① $S^\dagger \tau_j S=\tau_j$. $S^\dagger \sigma_j S=\sigma_j \omega$

$S^\dagger H_0 S=H_0$

② $C\tau_j C=\tau_j^\dagger$. $C\sigma_j C=\sigma_j^\dagger$

$C H_0 C=H_0$

2) duality $\begin{cases} (\sigma_i^+\sigma_{i+1}) \longrightarrow \tau_{i+1} \\ \tau_i \longrightarrow (\sigma_i^+\sigma_{i-1}) \end{cases}$

$H_0(J,h) \xrightarrow{\text{duality}} H_0(h,J)$

if $(J=h)$

Q	$Q=4$	$Q=5$	$Q=6$	$Q=7$
S	∞	~ 2.512	~ 1.48	~ 4.8

1973 Baxter

1) $Q\le 4$ → continuous

2) $Q>4$ → first-order

phase diagram

$h<J$ SSB

sym $h>J$ phase

$h=J$

h/J

RG

(Cardy 1980)

$Q \leq 4$

ϵ – thermal op. ($\Delta_\epsilon < 2$)

σ – ordering op. ($\Delta_\sigma < 2$)

ϵ' — dilution op. ($\Delta_{\epsilon'} \sim 2$)

$$H = H_{FP} + g_{\epsilon'} \int d^2x\, \epsilon'(x) + \cdots$$

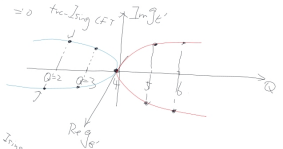
$$\frac{dg_{\epsilon'}}{dl} = a(Q-4) + g_{\epsilon'}^2$$

$$g_{\epsilon'}^* = \pm\sqrt{a(4-Q)} \quad (a>0)$$

$$\frac{dg_t}{dl} = \pm\left[(\Delta_\epsilon - 2) + h g_{\epsilon'}\right] g_t$$

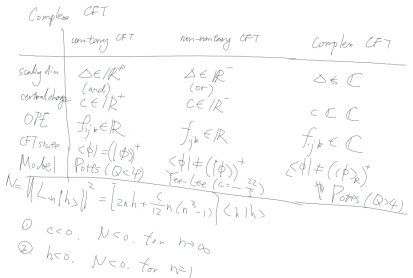
$$\frac{dg_\sigma}{dl} = \left[(\Delta_\sigma - 2) + e\, g_{\epsilon'}\right] g_\sigma$$

$$\Rightarrow g_\sigma = 0,\ g_\epsilon = 0$$



$Q > 4$

$$g_{\epsilon'}^* = \pm i\sqrt{a(Q-4)}$$ Ising CFT


Complex CFT
unitary CFT
non-unitary CFT
Complex CFT
scaling dim
$\Delta \in \mathbb{R}^+_0$
(and)
central charge
$c \in \mathbb{R}^+$
OPE
$f_{ijk} \in \mathbb{R}$
CFT states
$\langle\phi| = (|\phi\rangle)^+$
Models
Potts ($Q<4$)
$\Delta \in \mathbb{R}^-$
(or)
$c \in \mathbb{R}^-$
$f_{ijk} \in \mathbb{R}$
$\langle\phi| \neq (|\phi\rangle)^+$
for Lee ($c=-\frac{22}{7}$)
$\Delta \in \mathbb{C}$
$c \in \mathbb{C}$
$f_{ijk} \in \mathbb{C}$
$\langle\phi| \neq (|\phi\rangle_k)^+$
$\forall$ Potts ($Q>4$)
$N = \| L_{-n}|h\rangle \|^2 = \left[2nh + \frac{c}{12} n(n^2-1)\right] \langle h|h\rangle$
① $c<0$. $N<0$. for $n \to \infty$
② $h<0$. $N<0$. for $n=1$

A non-hermitian Potts model

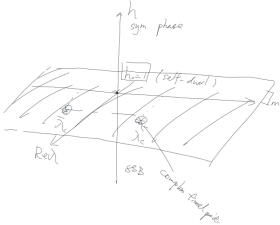
$H = H_0(J,h) + H_1(\lambda,\dots) \qquad (Q=5)$

$H_0 = -\sum_{i=1}^{L}\sum_{k=1}^{Q-1}\left(J\,(\sigma_i^+\sigma_{i+1}^-)^k + h\,\tau_i^k\right)$

$H_1 = \lambda \sum_{i=1}^{L}\sum_{k_1,k_2=1}^{Q-1}\Big(\left(\tau_i^{k_1}+\tau_{i+1}^{k_1}\right)(\sigma_i^+\sigma_{i+1}^-)^{k_2} + (\sigma_i^+\sigma_{i+1}^-)^{k_1}\left(\tau_i^{k_2}+\tau_{i+1}^{k_2}\right)\Big)$

	duality	S_Q sym	fermion
H_0	Y ($J=h$)	Y	Y
H_1	Y	Y	N ($\lambda \in \mathbb{C}$)

$\begin{cases} (\sigma_i^+\sigma_{i+1}^-) \to \tau_{i+1} \\ \tau_i \to \sigma_i^+\sigma_{i+1}^- \end{cases}$ ⊙ $\tau_i^{k_1}(\sigma_i^+\sigma_{i+1}^-)^{k_2} \to (\sigma_i^+\sigma_{i+1}^-)^{k_1}\tau_{i+1}^{k_2}$



$E_n - E_0 = \frac{2\pi\Delta_n}{L} + \frac{\alpha_n}{L^3} + \dots$

$\frac{E_n - E_0}{E_1 - E_0} \sim \Delta_n$

conformal tower

"emergent conformal sym"

	op.	Δ_n (model)	Δ_n (AC)
"h" →	ϵ	$0.466 - 0.224i$	$0.463 - 0.224i$
"λ" →	ϵ'	$1.908 - 0.599i$	$1.900 - 0.598i$
	ϵ''	$4.328 - 1.123i$	$4.340 - 1.135i$
→	σ	$0.134 - 0.021i$	$0.133 - 0.021i$
	⋮		

$\epsilon \sim (\sigma_i^t \sigma_{i+1})^k \ominus \tau_t^k$

dual-odd

ϵ' dual-even

S_2-even

More examples

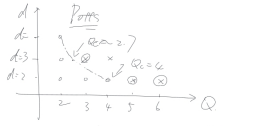
① 2D $Q=6$

② 3D $Q=3$.

"Fuzzy sphere"

③ 2+1D SO(5) DQCP. Néel–VBS

Fuzzy sphere.



	σ Δ_σ	ϵ Δ_ϵ	ϵ' $\Delta_{\epsilon'}$	Δ_{VB}	Δ_{NN}
Fuzzy sphere	0.685	1.458	2.845	2	3.8
SO(5) CFT	0.630	1.519	2.857	2	3.884
J-Q model	0.607	1.491	2.27	2.0	3.723.