Renormalized Conformal Perturbation Theory (CPT) from String Field Theory (SFT)

based on Mazel, Sandor, C. Wang, XY 2403.14544
and W.I.P.

[earlier work: Mukherji, Sen '91, ...]

"Name" CPT

$$\text{CFT} + \Delta S$$

$$\Delta S = \int dx\, \phi(x)$$

relevant or marginal

Insert $e^{-\Delta S}$ into correlator

defined w/ UV regulator.

Wilsonian renormalized CPT

Insert

$$\left[e^{-\int dx\, \phi(x)} \right]_{reg}$$

linear combo of all (scalar) op's

$$\equiv \sum_{n=0}^{\infty} \frac{(-)^n}{n!} \int_{V_n} dx_1 \cdots dx_n \, \phi(x_1) \cdots \phi(x_n)$$

suitable domain e.g. $|x_i - x_j| > r$

floating cutoff

RG transf

$$V_n(r) \longrightarrow V_n(r+\delta r)$$

$$\phi \longrightarrow \phi + \delta\phi$$

built out of ops

$$\phi = \sum t_k \phi_k$$

s.t. $\left[\ \cdots\ \right]_{reg}$ remains invariant

A 2d example

(compact)

CFT $\mathcal{M}$ $\otimes$ (n free noncompact bosons X_i)

$\Downarrow$

a pair of marginal primaries ϕ_1, ϕ_2

$$\phi_1 \times \phi_1 \sim 1 + \phi_2 + \text{irr.}$$

$$\phi_1 \times \phi_2 \sim \phi_1 + \text{irr.}$$

$$\phi_2 \times \phi_2 \sim 1 + \text{irr.}$$

prototype:

$\mathcal{M}$ = compact boson

$\phi_1 = \underline{\cos(\sqrt{2}X)}$

$\phi_2 = \underline{\partial X \bar{\partial} X}$

Consider deformation

$$\Delta S = \int d^2z \left[f(\vec{X})\, \phi_1 + g(\vec{X})\, \phi_2 \right]$$

new fixed pt?

Naive CPT: 1^{st} order marginal: $\nabla^2 f = \nabla^2 g = 0$

In renormalized CPT: $\nabla^2 f + 2 f g = 0$

"Horowitz-Polchinski string star" $\nabla^2 g + f^2 = 0$

Embed 2d CFT into worldsheet

"matter"

$$\boxed{\mathbb{A} \oplus \mathbb{M}} \oplus (b\,c \text{ ghost})$$

$c_{\mathbb{A}}$

auxiliary "Mock" CFT

$c_{\mathbb{M}} = 26 - c_{\mathbb{A}}$

$c_{gh} = -26$

<u>ghost</u>

$$b_{zz}(z), \quad c^{z}(z), \quad \tilde{b}_{\bar z \bar z}(\bar z), \quad \tilde{c}^{\bar z}(\bar z)$$

$$(2,0) \qquad (-1,0) \qquad (0,2) \qquad (0,-1)$$

$$b(z)\,c(0) \sim \frac{1}{z} + \text{reg.} \qquad T^{gh} = -(\partial b)c - 2b\partial c.$$

ghost $U(1)$

- ghost # assigns $+1$ to $c, \tilde{c}$

  -1 to $b, \tilde{b}$

- anomaly: $\left\langle c(z_1)c(z_2)c(z_3)\tilde{c}(\bar{z}_1)\tilde{c}(\bar{z}_2)\tilde{c}(\bar{z}_3)\right\rangle = |z_{12}z_{13}z_{23}|^2$

BRST $\quad Q_B = \oint \frac{dz}{2\pi i} j_B(z) + \text{c.c.}$

Key properties:

$Q_B^2 = 0$

$Q_B \cdot b = T = T^m + T^{gh}$

$(1,0)$ primary $\rightarrow$ $j_B = c\,T^{\text{matter}} + bc\partial c + \frac{3}{2}\partial^2 c$

"String field" space

$$\hat{\mathcal{H}} = \{ \Psi \in \mathcal{H}^m \otimes \mathcal{H}^{gh} : b_0^- \Psi = L_0^- \Psi = 0 \}$$

e.g.

$$\Psi = \underbrace{c\tilde{c}}_{\text{lowest weight state in } \mathcal{H}^{gh}} O_1^m + \dots$$

$(-1, -1)$

$$b_0^\pm \equiv b_0 \pm \tilde{b}_0$$

$$L_0^\pm \equiv L_0 \pm \tilde{L}_0$$

$$b(z) = \sum_{n=-\infty}^{\infty} \frac{b_n}{z^{n+2}}$$

(classical, bosonic, closed) SFT equation of motion

$$Q_B \Psi + \sum_{n=2}^{\infty} \frac{1}{n!} [\Psi^{\otimes n}] = 0$$

"n-string bracket"

change ghost # by $-2n+3$ $\longrightarrow$ $[\cdot] : \hat{\mathcal{H}}^{\otimes n} \longrightarrow \hat{\mathcal{H}}$

symmetric, n-linear.

e.g. if Ψ has ghost # $= 2$, EoM has gh # $= 3$

Key property of $[\cdot]$: L_∞-algebra

$$Q_B[\Psi^{\otimes n}] = -n\left[Q_B\Psi \otimes \Psi^{\otimes(n-1)}\right] - \sum_{\ell=1}^{n-2} \binom{n}{\ell} \left[\Psi^{\otimes \ell} \otimes \left[\Psi^{\otimes(n-\ell)}\right]\right]$$

"background independence"

$$\begin{tikzcd}
 & \mathbb{A} \arrow[r, "\text{deform}"] \arrow[d, Leftrightarrow] & \mathbb{A}_\lambda \\
\mathrm{SFT}_{\mathbb{A}} & \Psi = 0 \arrow[r] & \Psi = \Psi_\lambda
\end{tikzcd}$$

nontrivial sol'n to SFT EOM

$$[\Psi^{\otimes n}] = \frac{1}{(2\pi i)^{n-2}} \, b_0^- \mathbb{P}^- \int_{\Gamma_{n+1}} e^{\mathcal{B}} \prod_{i=1}^{n} \Psi^{(f_i)}$$

$b_0 - \tilde{b}_0$

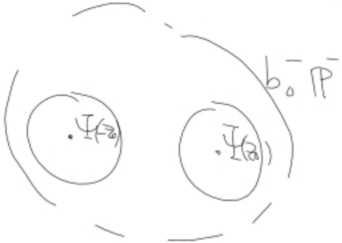
projection onto $L_0^- = 0$

$\Gamma_{n+1} \subset \hat{\mathcal{M}}_{0,n+1}$

f_i: conformal transf

$\mathcal{B} \sim \oint b \cdots$

$$[\Psi^{\otimes 2}] =$$



"flat-vertex frame"

Another example

$\mathbb{A}$ = free noncompact boson X

$$A \oplus D_\beta \oplus M$$

$$A_\lambda \oplus D_{\beta(\lambda)} \oplus M$$

consider $\Delta S = \lambda \int d^2z \, \cos(\sqrt{2}X)$

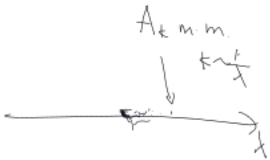
Claim: $\exists$ marginal deformation to all-orders in λ (perturbatively)

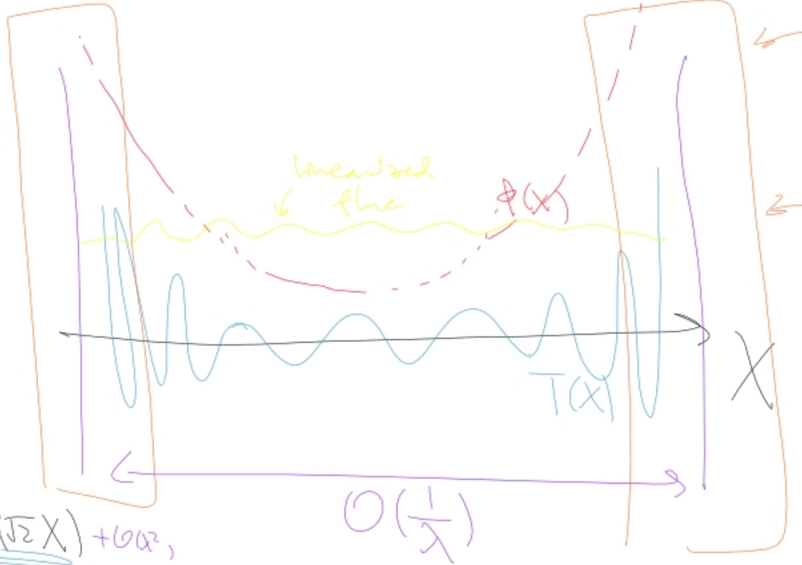
SFT: $\Psi = \sum_{n=1}^{\infty} \lambda^n \Psi_n$ solves EOM order by order in λ

- $\Psi_1 = e\tilde{c}\,\cos(\sqrt{2}X)$

- To solve $G_B \Psi_2 = \cdots$

  - encounter obstruction $\rightsquigarrow$ $\Delta c_A = O(\lambda^2)$
  - remove obstruction by including a linear dilaton sector in M. & turn on $\Delta\Psi$





"wall"

$\lambda \to 0, \quad g \to 1$

Runkel-Watts

$\left(\lim_{k\to\infty} A_k \text{ min. model} \right)$

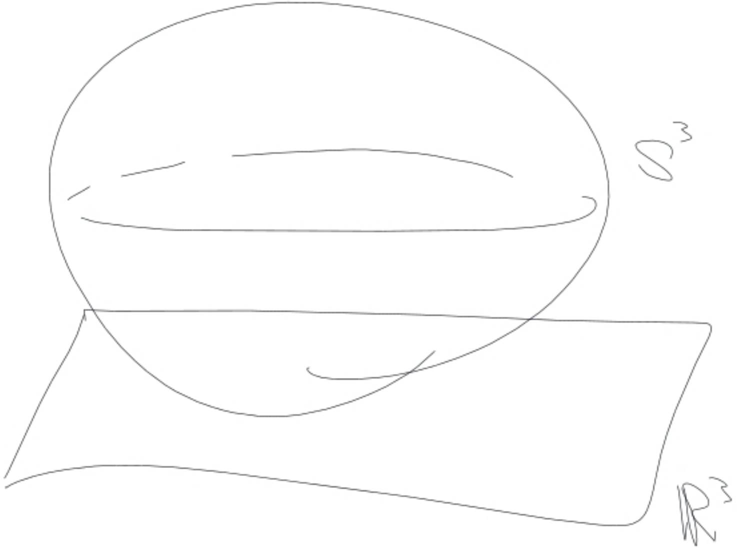
$T(X) \overset{\lambda}{\propto} \cos(\sqrt{2}X) + O(\lambda^2)$,

A_k min. model

$k \sim \frac{\#}{\lambda}$

dilaton $\phi(X) \longleftrightarrow$ NLσM

$\Delta S = \int \phi(x) R(g)$



(w/ H-flux)

$R \sim \sqrt{k}$

$\frac{1}{R}$ expansion