Gravity from Matrix Quantum Mechanics

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Outline

I Intro – BFSS conjecture

II The BMN model

III Strong coupling EFT of BMN

IV Bootstrap (a bit)

Ref for I:

→ • Maldacena Strings 2024

→ • Xi S-mat bootstrap 2024

• Polchinski '99

• Itzhaki, Maldacena, Sonnenschein, Yankielowicz '98 canonical ex:

N D3 branes → $\mathcal{N}=4$ SYM (3+1) d max susy

→ IIB $AdS_5 \times S^5$

today:

N D0 → $H_{BFSS} = \Lambda\, \mathrm{Tr}\left(\frac{1}{2}P_I^2 - \frac{1}{4}[X^I, X^J]^2 - \frac{1}{2}\psi_\alpha \gamma^I_{\alpha\beta}[X^I, \psi_\beta]\right)$ $\quad \Lambda = g_{YM}^{2/3}$

$[g_{YM}^2] = [M^3]$ $\quad P_I \quad X^I \quad \psi_\alpha$ N×N hermitian

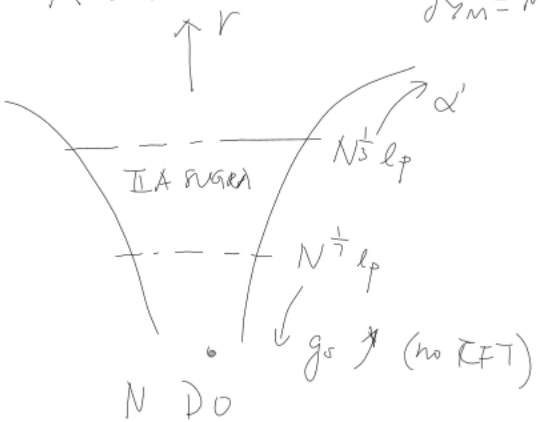
max susy in (0+1) d

→ IIA / M-theory

S^1 with $\tilde{R}$ $\qquad g_s = (\tilde{R}/l_p)^{3/2}$

AdS"/"CFT" pic

$g_{YM}^2 = M^6 R^3$



uplifted from IIA to 11d

$$ds_{11}^2 = -2\,d\underset{\text{'time'}}{x^+}dx^- + d\vec{x}^2 + \boxed{\frac{\# N l_p^9}{R^2 r^7}(dx^-)^2}$$

$x^- \sim x^- + 2\pi R$ $\qquad r = |\vec{x}|$ 9-dim

$r \to \infty$ flat? stringy

$r \to 0$

BFSS conj: $T_{\underline{\text{M-theory}}} = \lim_{\substack{R\to\infty \\ N\to\infty \\ -P_- = \frac{N}{R} \text{ fixed}}} T_{BFSS}$

$$\frac{P_- l_p^9}{R r^7}(dx^-)^2$$

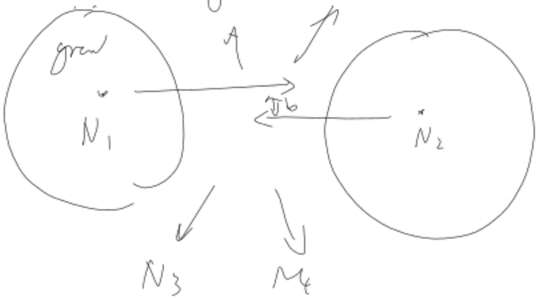
$$\frac{r}{l_p} \gg \left(\frac{R}{l_p}\right)^{-\frac{1}{7}}$$

difference: beyond AdS/CFT

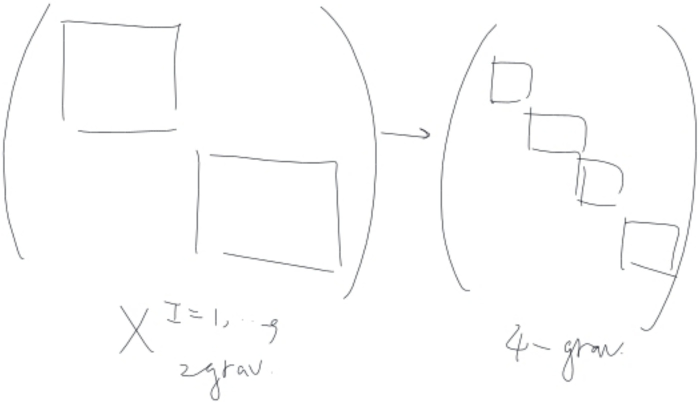
Flat space holography ($\neq$ F.S.L.)

($\neq$ celestial)

Scattering Pic



$$-P_-^{\,i} = \frac{N_i}{R} \xrightarrow{\text{mom. conserv}} \sum_a N_a^{(i)} = \sum_b N_b^{(f)}$$



features of BFSS MQM

$SO(9) \times SU(N)$ $\quad \mathcal{N}=16$

- susy → flat directions → scattering st.

continuous spec.

- unique G. St. (normalisable)

Also, MQM is simpler than QFT

II BMN model

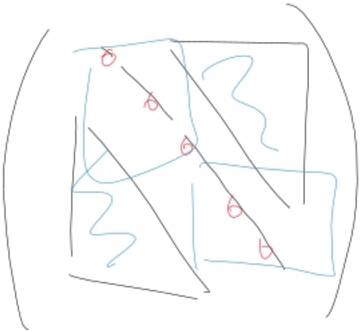
$$H_{BMN} = H_{BFSS} + \mathrm{Tr}\left[g_{YM}^{-2/3} \frac{1}{2} \left(\frac{\mu}{3}\right)^2 (X^i)^2 + g_{YM}^{-2/3} \left(\frac{\mu}{6}\right)^2 X^a \right.$$

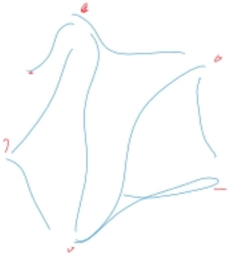
$$+ \mu(\cdots) + \mu(\cdots)$$

$[\mu] = [M]$

interesting handles on BFSS $\longrightarrow$ $g^2 = \dfrac{g_{YM}^2}{\mu^3}$ dim less

- BMN = BFSS in a box $\longrightarrow$ discrete spec
  better for numerics
- Susy loc ~~applic~~ applicable





X^I

N_i $D0 \longrightarrow$ graviton $\quad -P_-^i = \frac{N_i}{R}$

rich by itself

- $SO(3) \times SO(6)$
- $\underline{\underline{p(N)}}$ ~~vac~~ vacua

$N = 16$ $\qquad i = 1, 2, 3$

$$V_{bos} = \mathrm{Tr}\left[\left(\frac{1}{3} X^i + i\epsilon^{ijk} X^j X^k\right)^2\right.$$
$$\left. + \left(\quad\right)^2 + \left(\quad\right)^2\right]$$

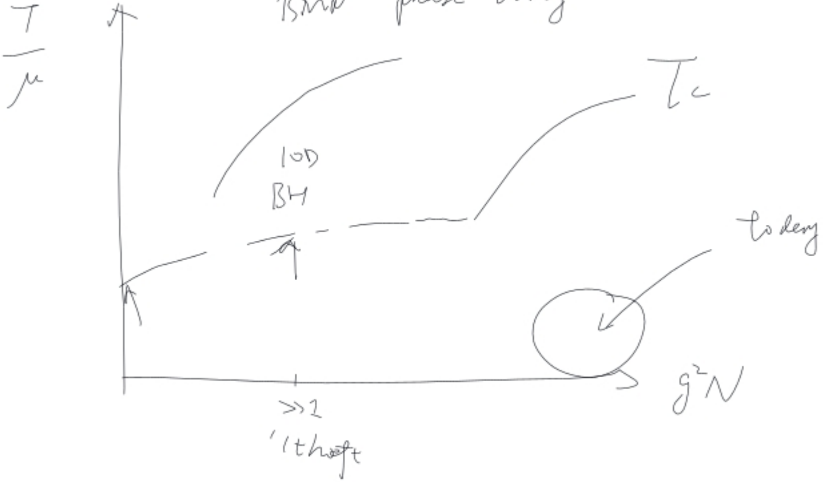
$X^p \qquad p = 4 \cdots 9$

$$X^i = \frac{1}{3g} J^i \qquad [J^i, J^j] = i\epsilon^{ijk} J^k$$

$$X^p = 0$$

$$\begin{pmatrix} \square & & \\ & \square & \\ & & \square \end{pmatrix}$$

BMN phase diag



BMN conj:

M-thy in 11D pp-wave $= \lim_{\substack{R\to\infty \\ N\to\infty}}$ BMN

$$g^2 = \frac{g_{YM}^2}{\mu^3} = \frac{R^3}{\mu^3 l_p^6}$$

$\frac{N}{R}$ fixed

μl_p fixed $\longrightarrow$ g^2/N^3 fixed

pp-wave $ds^2 =$ flat $+ \left(\frac{\mu}{3} (x^i)^2 + \frac{\mu}{6} (x^p)^2 \right) (dx^+)^2$

$i = 1, 2, 3$ $\quad$ $p = 4, \ldots, 9$

$\Downarrow$

a box $\longleftrightarrow$ harmonic pot.

Today BMN @ $g \to \infty$ general N

$$g^2 = \frac{R^3}{\mu^3 l_p^6} \to \infty \Rightarrow l_p \to 0 \longrightarrow \text{SUGRA}$$

$$-\frac{g^2}{4}\,\mathrm{Tr}\,[X^I, X^J]^2$$

$$X^I = d^I + W^I$$

diag off-diag

slow fast ← integrate

result:
$$H_{BMN} = \sum_{a=1}^{N} \Big[\frac{\mu}{3} \underline{(b_a^i)^\dagger b_a^i} + \frac{\mu}{6} \underline{(c_a^p)^\dagger c_a^p} \qquad i=1\cdots 3 \quad p=4\cdots 9$$

$$+ \frac{\mu}{4} (\lambda_a)_\alpha^\dagger (\lambda_a)_\alpha \Big] \qquad \alpha = 1\cdots 8$$

$$+ O\big(\underline{\underline{1/g^3}}\big) \qquad b \sim X^i_{aa} + i P^i_{aa}$$

$H \quad O(g)$

$\downarrow$

$O(g^{1/2})$

$\downarrow$

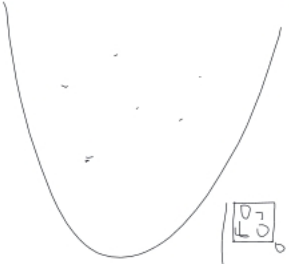
$\underline{O(g^0)}$ N decoupled Susy H.O.

$\downarrow$

Valid when $|X^1_{aa} - X^2_{bb}| \gg g^{-1/3}$ $(\longleftarrow)$

$$\{Q_\alpha, Q_\beta\} = \delta_{\alpha\beta} H + M^{ij} + M^{pq} + \underset{\substack{\uparrow \\ SU(N)}}{G}$$

harmonic pot



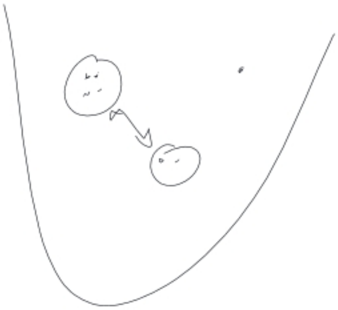
NxN

postulate

BMN bound states

$\longrightarrow$

$\to (N-1)\times(N-1)$



~~et~~ same spec.

$N \times N \to p(N)$ config

Why? if $\left| X^{1}_{aa} - X^{2}_{bb} \right| \sim g^{-1/3}$ no ~~fast~~ / slow

$\tilde{X} = X g^{\frac{1}{3}}$ $\qquad \tilde{p} = p g^{-\frac{1}{3}} \longrightarrow H_{BMN} = g^{\frac{2}{3}} H_{BFSS} + O(1)$

$\tilde{X} = O(g^{0})$

BMN B.S. $\sim$ BFSS b.s. slightly deformed

BMN spec $\begin{pmatrix} g \to \infty \\ \forall N \end{pmatrix}$ = SUGRA linearised spec in $\underline{11}$ pp-wave DLCQ

$-(P_-)^i$

pp-wave

$-P_-^i \sim N_i$ (size of BMN B.S.)

x

$x_1 \quad x_1 \quad x_2$

$|x_1 - x_2| = x$

$V \sim \dfrac{\alpha}{|x|+1}$