Thermal EFTs for CFTs

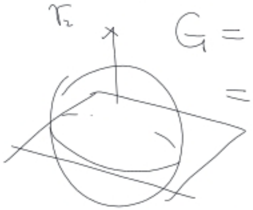
2405.17562

with N. Benjamin, J. Lee, S. Pal, D. Simmons-Duffin

- Thermal EFTs

$$\mathrm{Tr}_{\mathcal{H}(S^{d-1})}\left[e^{-\beta(H - i\vec{\Omega}\cdot\vec{J})}\right], \qquad S^1_\beta \times S^{d-1}_L, \text{ twist by } \beta\vec{\Omega}$$

Couple my CFT to the metric



$$G = ds^2 = d\tau^2 + \underbrace{\sum_{a=1}^{\lfloor d/2\rfloor+\epsilon} dr_a^2 + \sum_a r_a^2 d\theta_a^2}_{ds^2_{\text{sphere}}}, \quad (\tau, \theta_a) \sim (\tau+\beta, \theta_a - \beta\Omega_a)$$

$$= g_{ij}dx^i dx^j + e^{2\phi}(d\tau + A_i dx^i)^2$$

$S^{d-1} \subset \mathbb{R}^d$ $\quad \sum_a r_a^2 = 1$

$z_1 = r_1 e^{i\theta_1}, \; z_2 = r_2 e^{i\theta_2}$

$$Z_{CFT}[G] = Z_{QFT}[g, A, \phi] \sim e^{-S_{th}[g,A,\phi]}$$

agree up $e^{-\# L/\beta}$ → non-pert corrections

(1) $(d-1)$-dim coordinate invariance + gauge invariance

(2) d-dim Weyl invariance $\quad Z_{CFT}[e^{2\sigma}G] = Z_{CFT}[G]\,e^{-S_{\text{anom}}}$

$$-\log \mathrm{Tr}\left(e^{-\beta(H - i\vec{\Omega}\vec{J})}\right) \sim S_{th} = \int d^{d-1}x \, \frac{\sqrt{\hat{g}}}{\beta^{d-1}} \left(\underline{-f} + c_1 \beta^2 \underline{\hat{R}} + c_2 \beta^2 \underline{F^2} + \cdots \right), \quad \underline{\hat{g} = e^{-2\phi} g} \qquad F = dA$$

$$= \frac{\mathrm{Vol}(S^{d-1})}{\prod_{i=1}^{\lfloor d/2 \rfloor} (1+\Omega_i^2)} \left[-\frac{f}{\beta^{d-1}} + (d-2)\left(\boxed{(d-1)c_1 + \left(2c_1 + \frac{8}{d} c_2\right) \sum_{i=1}^{\lfloor d/2 \rfloor} \Omega_i^2} \right) \frac{1}{\beta^{d-3}} + \cdots \right]$$

M_{d-1} $\quad \mathcal{H}(M_{d-1})$ $\quad \overline{\mathrm{Tr}}_{\mathcal{H}(M_{d-1})} \left(e^{-\beta(H - \vec{\xi}\,\vec{Q})} \right) \sim e^{\underline{-S_{th}}}$

$R = e^{i\vec{\theta}\vec{J}}$, $\quad \theta_i = \frac{2\pi p_i}{q_i}$ $\qquad \cancel{\beta\vec{\Omega} = \vec{\theta}}$

$$\frac{1}{\beta^{d-1+k}} \left(\Omega_i^k + \cdots \right)$$

(d=2) $\quad S_{th} = -\dfrac{2\pi \tilde{f}}{\beta(1+\Omega^2)}\ , \quad E_{vac} = -f/\beta \Rightarrow f = \dfrac{2\pi c}{12}$

$$Z_{2d\,CFT} = \underbrace{\exp\left(\frac{4\pi^2 c L}{12\beta(1+\Omega^2)}\right)}_{\text{contribution from ground states in modular dual channel}} + \underbrace{e^{-\# L/\beta} + \cdots}_{\text{excited states}}$$

this can be reproduced from S-invariance of $Z(\tau, \bar{\tau})$!

$$\left.\begin{array}{l} - \ \rho^{d>2}_{A,J}(f, c_1, c_2, \cdots) \\ - \ C_{HHH} \end{array}\right\} \ 2306.08031$$

Today:

- $\mathrm{Tr}\left((-1)^J e^{-\beta H}\right)$ vs $\mathrm{Tr}\left(e^{-\beta H}\right)$ (Beyond S-transform)
- Non-pert. terms (Beyond vacuum states)

(d>2)

— Twisted Partition functions

$R \in SO(d)$, $R^q = 1$ e.g. $R = (-1)^J$. $R = e^{\frac{2\pi i}{5} J_1 + \frac{7\pi i}{3} J_2} \in SO(4)$

$q = 15$

- $\langle R \rangle$ acts freely on S^{d-1}

$$-\log \mathrm{Tr}\left[R e^{-\beta H}\right] \sim -\frac{1}{q} \log \mathrm{Tr}\left[e^{-q\beta H}\right] + \text{" Change in topological terms "}$$

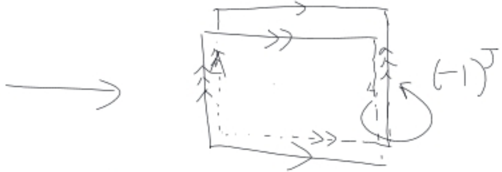
- $\langle R \rangle$ action is not free $R = e^{2\pi i p\sigma/q} \in SO(3)$

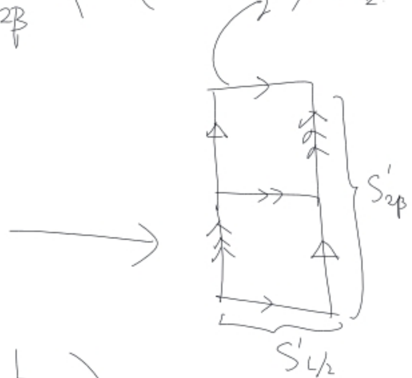
$$-\log \mathrm{Tr}\left[R e^{-\beta H}\right] \sim -\frac{1}{q} \log \mathrm{Tr}\left[e^{-q\beta H}\right] + \text{" Change in topological terms "}$$

$$+ \text{" defect actions "}$$

– Folding Trick change in topo. terms $\Longleftarrow \; (S^1_L/\mathbb{Z}_2)\times S^1_{2\beta} \neq (S^1_L\times S^1_{2\beta})/\mathbb{Z}_2$

$\mathrm{Tr}\left(\underline{(-1)^J} e^{-\beta H}\right)$ in a 2d CFT





S^{d-1}
M_{d-1}

$$\mathrm{Tr}\left((-1)^J e^{-\beta H}\right) \sim e^{-\frac{1}{2}S_{th}[2\beta]} = \exp\left(\frac{\pi^2 c L}{12\beta}\right)$$

This also follows from modular invariance under $\begin{pmatrix} -1 & 0 \\ 2 & -1 \end{pmatrix}$!

$$\mathrm{Tr}\left(e^{\frac{2\pi i p J}{q}} e^{-\beta H}\right) \sim e^{-\frac{1}{q}S_{th}(q\beta)}$$

- Higher d CFTs: no modular invariance, folding trick works!

$$-\log \mathrm{Tr}_{\mathcal{H}(M_{d-1})}\left(R e^{-\beta H}\right) \sim \int_{M_{d-1}/\langle R\rangle} \frac{\sqrt{\hat{g}}}{(q\beta)^{d-1}}\left(-f + \cancel{c_1 (q\beta)^2 \hat{R} + \cdots}\right)$$

$$\sim -\frac{1}{q}\log \mathrm{Tr}_{H(M_{d-1})}\left(e^{-q\beta H}\right) \qquad R^q = 1$$

- The effective free energy will always be rescaled by q^{-d}

- Defect action

$S^{d-1}/\langle R \rangle$ is not smooth $\Longrightarrow S_D$

- Contribution from S_D is always subleading $\dfrac{\tilde{f}}{\beta^D}$, $D = \dim(\text{defect})$

→ Example: 3d CFT $\mathrm{Tr}\left(\underbrace{e^{2\pi i pJ/q}}\, e^{-\beta H}\right)$

$$S_{D_n} + S_{D_s} = \frac{a_{0,p/q} + a_{0,-p/q}}{q} + 2\beta\Omega\left(\cancel{\frac{a_{1,p/q} - a_{1,-p/q}}{}}\right) + \cdots$$

$$\frac{a_{i,x} = a_{i,x+1} \quad \text{bosonic}}{p/q = 1/2}$$